22C:34 — Spring 2004

Homework II

1. [15 points]
In a round-robin tournament (e.g., tennis, chess, etc.) each participant plays every other participant one match. Use induction to show that in a round-robin tournament with $n \geq 2$ participants, the number of matches, RR(n), is $RR(n) = \frac{n*(n-1)}{2}$.

2. [15 points]
Problem 11, p.132 of our text (note that this problem pertains to the “fpes” defined on page 15).

3. [20 points]
Prove by induction that for all natural numbers n, $0 \cdot 1 + 1 \cdot 2 + 2 \cdot 3 + \ldots + n \cdot (n+1) =$

$$\sum_{k=0}^{n} k \cdot (k+1) = \frac{2n^3 + 6n^2 + 4n}{6}$$

4. [20 points]
Using lists as defined on p. 145 of our text, we define a function 'cat' taking two lists as arguments and returning a list as follows (for all items a and lists x and y):

cat([ ], y) = y,
cat (a.x, y) = a.cat(x,y).

Prove that $cat([a_1, a_2, \ldots, a_n], [b_1, b_2, \ldots, b_m]) = [a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_m]$ for all $m,n \geq 0$.

5. [5 points]
Problem 13, p. 243 of our text.

6. [20 points]
Problem 15, p. 243 of our text.