22C:185 — Fall 2004

Homework VIII

1 (corrected). [20 points]
The Fibonacci numbers are defined by fib(0) = 0, fib(1) = 1, and fib(n) = fib(n-1) + fib(n-2) for n≥2. Consider the recursive program over all integers Z

$\mathcal{P}$: f(n) = **if** n=0 **or** n=1 **then** n **else** f(n-1) + f(n-2).

(a) Show that the Fibonacci function fib (i.e., the partial function that is undefined for arguments less than 0) is a fixed point of the corresponding fixed point functional $\mathcal{P}$,

$\mathcal{P}$(g) (n) = **if** n=0 **or** n=1 **then** n **else** g(n-1) + g(n-2).

(b) Show that the total function h: Z-->Z is also a fixed point of $\mathcal{P}$, where h is defined by

h(n) = fib(n) if n≥0,
h(n) = fib(-n) if n<0 and n odd,
h(n) = -fib(-n) if n<0 and n even.

2. [25 points]
Determine the least fixed point of the functional associated with the recursive function below (over all integers Z) and justify your answer.

f(n) = **if** n>7 **then** n-5 **else** f(f(n+6))

3. [20 points]
Show that the following functions are continuous:

(a) or: Bool□Bool --> Bool (i.e., logical or), where Bool and Bool□Bool are the pointed cpos from our text.

(b) ≥: Nat□Nat --> Bool, where Nat, Nat□Nat and Bool are the pointed cpos from our text.