Homework I

1. [15 points]
Taking $\{0,1\}$ and start = $S$ in the BNF

$S ::= 0B | 1A$
$A ::= 0 | 0S | 1AA$
$B ::= 1 | 1S | 0BB$

show derivation trees for each of the following
(a) 001011
(b) 10111001
(c) 10011100

2. [10 points]
Taking $\{a,b\}$ and start = $S$ in the BNF below, show two distinct derivation trees for the string $aab$.

$S ::= aSB | \epsilon$
$B ::= bB | \epsilon$

3. [10 points]
Consider the two grammars $G_1$ (start=W) and $G_2$ (start=Y) below with $\{a,b,c\}$. Determine whether or not they are equivalent (i.e., $L(G_1) = L(G_2)$) and prove your answer.

<table>
<thead>
<tr>
<th>$G_1$</th>
<th>$G_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W ::= WC</td>
<td>X$</td>
</tr>
<tr>
<td>$X ::= aXb</td>
<td>\epsilon$</td>
</tr>
</tbody>
</table>

4. [20 points]
Determine whether each of the following identities is true for all languages $L_1, L_2 \subseteq \{a,b,c\}^*$, and justify your answers.

(a) $(L_1 \cap L_2)^* = L_1^* \cap L_2^*$
(b) $(L_1 \cdot L_2)^* = L_1^* \cdot L_2^*$
(c) $\varnothing^* = \varnothing$
(d) $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$