Homework I

1. [10 points]
   Show that for any wffs $a$, $b$ and $g$, the propositional formula $(\neg b \lor \neg g) \lor \neg \neg \neg a \lor \neg g$ is a tautology.

2. [15 points]
   Provide an expression utilizing only the 'nand' operation (negated 'and', see truth table definition below) that is logically equivalent to each of the three usual Boolean operations $\lor$, $\land$, and $\neg$.

   \[
   \begin{array}{c|c|c}
   P & Q & P \text{ nand } Q \\
   \hline
   T & T & F \\
   T & F & T \\
   F & T & T \\
   F & F & T \\
   \end{array}
   \]

3. [10 points]
   Show that $(x \text{ nand } y) \text{ nand } z$ is not logically equivalent to $x \text{ nand } (y \text{ nand } z)$ (i.e., nand is not associative).

4. [10 points]
   Determine if the program assertion (see Chapter 14 of Diller) below is valid and justify your answer. Assume that the domain of the program variables is integers.
   \[
   \{\text{true}\} \text{ if } X>Y \text{ then skip else } X:= X\times\times Y \{X>Y\}
   \]