

## Homework I

### 1. [15 points]

Take character set  $C = \{a, b\}$ , and languages  $S = \{b, aba\}$  and  $T = \{\epsilon, bb\}$ .

- (a) what is the language  $T^2$ ?
- (b) what is the language  $T^*$ ?
- (c) what is the language  $S \cdot T^*$ ? [note: this means  $S \cdot (T^*)$  not  $(S \cdot T)^*$ ]
- (d) what is the language  $(S \cdot T)^*$ ?
- (e) what is the language  $(S^*)^*$ ?

### 2. [20 points]

Take character set  $C = \{a, b\}$ , and consider languages  $R = \{a^{2i}\}^*$  (infinite -- all even nos. 'a'),  $S = \{a^{2i+1}\}^*$  (infinite -- all odd nos. 'a'). Justify your answers to each of the following questions:

- (a) what is the language  $R \cup S$ ?
- (b) what is the language  $R \cdot S$ ?
- (c) what is the language  $R^*$ ?
- (d) what is the language  $S^*$ ?

### 3. [30 points]

For  $C = \{a, b\}$ , write a regular expression that describes each of the languages below, and justify that your answer describes exactly the required strings -- these are set equality demonstrations, every required string must be described by your regular expression, and *no* other string can be.

- (a)  $\{\epsilon, a^3, a^5, a^6, a^9, a^{10}, a^{12}, a^{15}, \dots\}$  -- all and only strings of 'a's whose length is either a multiple of 3 or a multiple of 5
- (b) all and only strings that either (i) begin with 'aa' and have no 'a' following a 'b', or (ii) begin with 'bb' and have no 'b' following an 'a'
- (c)  $\{a, a^2, a^4, a^5, a^7, a^8, \dots\}$  -- all and only strings of 'a's whose length is not a multiple of 3.

### 4. [15 points]

For the BNF definition  $X ::= b \mid aaaXa \mid aaXaa$ , determine whether or not the string below is in  $L(X)$ , and justify your answer — either a derivation if yes, or what makes it impossible if no.

- (a)  $a^7ba^5$
- (b)  $a^6ba^4$
- (c)  $a^8ba^4$

**5. [20 points]**

Given the BNF definition  $X ::= \epsilon \mid XaXbX$ , determine whether or not each of the following strings are in  $L(X)$  and justify your answer -- either a derivation if yes, or what makes it impossible if no.

- (a) aabbab
- (b) abaaab
- (c) abbaab
- (d) aaabbb