Homework I

1. [15 points]
Take character set C = \{a, b\}, and languages S = \{b, aba\} and T = \{\epsilon, bb\}.
(a) what is the language \( T^2 \)?
(b) what is the language \( T^* \)?
(c) what is the language \( S \cdot T^* \)? [note: this means \( S \cdot (T^*) \) not \( (S \cdot T)^* \)]
(d) what is the language \( (S \cdot T)^* \)?
(e) what is the language \( (S^*)^* \)?

2. [20 points]
Take character set C = \{a, b\}, and consider languages R = \{a^2\}^* (infinite -- all even nos. 'a'), S = \{a\} \cdot \{a^2\}^* (infinite -- all odd nos. 'a'). Justify your answers to each of the following questions:
(a) what is the language \( R \cdot S \)?
(b) what is the language \( R \cdot S \)?
(c) what is the language \( R^* \)?
(d) what is the language \( S^* \)?

3. [30 points]
For C = \{a, b\}, write a regular expression that describes each of the languages below, and justify that your answer describes exactly the required strings -- these are set equality demonstrations, every required string must be described by your regular expression, and no other string can be.
(a) \{\epsilon, a^3, a^5, a^6, a^9, a^{10}, a^{12}, a^{15}, \ldots \} -- all and only strings of 'a's whose length is either a multiple of 3 or a multiple of 5
(b) all and only strings that either (i) begin with 'aa' and have no 'a' following a 'b', or (ii) begin with 'bb' and have no 'b' following an 'a'
(c) \{a, a^2, a^4, a^5, a^7, a^8, \ldots \} -- all and only strings of 'a's whose length is not a multiple of 3.

4. [15 points]
For the BNF definition \( X ::= b \mid aaaXa \mid aaXaa \), determine whether or not the string below is in \( L(X) \), and justify your answer — either a derivation if yes, or what makes it impossible if no.
(a) \( a^7ba^5 \)
(b) \( a^6ba^4 \)
(c) \( a^8ba^4 \)
5. [20 points]
Given the BNF definition $X ::= [] | XaXbX$, determine whether or not each of the following strings are in $L(X)$ and justify your answer -- either a derivation if yes, or what makes it impossible if no.
(a) aabbab
(b) abaaab
(c) abbaab
(d) aaabbb