

sample

Final Exam
Open book/notes

1. [25 points]

Consider the BNF definition $X ::= \epsilon \mid XaXbX$. For each of the strings below, show a derivation (or parse) tree if the string is in the language $L(X)$, and if not, briefly explain why.

- (a) ababab
- (b) abbaab
- (c) aababb
- (d) aabbab

2. [25 points]

Consider the Haskell functions f and g defined below.

- (a) what is the type of f ?
- (b) what is the type of g ?
- (c) what is the value of $f\ 95$?
- (d) what is the value of $g\ [1,2]$

```
f n
  | n>100    = n-10
  | n<=100   = f(f(n+11))

g [] = []
g [x] = [[ ], [x]]
g (x:xs) = g xs ++ [x:ys | ys<-g xs]
```

function definitions

3. [40 points]

Write a definition of an operation 'center', taking three arguments, a list, a non-negative integer, and a "fill" item, and whose result is a list of length given by the integer argument with the argument list "centered" in it (i.e., the number of fill items preceding and following should differ by at most one), and all other positions of the result containing the fill item. For instance (using Haskell notation) `center "abc" 6 '#'` should yield `"#abc##"` (or `"###abc#"`).

Provide a definition of the *function* 'center'. The definition can be written in Haskell or Prolog. Full credit will be awarded only if you state the language you use and provide justification that your program operates as required.

4. [40 points]

For the Prolog program given below, show *all* of the alternative responses to the query ?-p(X,[a,a,b,b]).

```
p(p1, [ ]).
p(p2(T1,T2), [a|Xs]) :-
    append(Ys,Zs,Xs), p(T1,Ys), q(T2,Zs).

q(q1, [ ]).
q(q2(T), [b|Xs]) :- q(T,Xs).

append([ ], Ys, Ys).
append([X|Xs], Ys, [X|Zs]) :- append(Xs,Ys,Zs).
```

predicate definitions

5. [40 points]

(a) [30 points]

Provide a Prolog definition of the *predicate* 'repeats(Xs,Ys)' that succeeds when Ys is the list of distinct items that appear more than once in list Xs (i.e., one occurrence in Ys of each repeated item in Xs). Your predicate should be able to solve for Ys given Xs. For instance, the query 'repeats([a,b,c,c,a], As)' should succeed with As=[a,c].

(b) [10 points]

Show *all* the alternative responses of your answer to part (a) for the query 'repeats([a,A,b], Bs)'.

Full credit will be awarded only if you provide justification that your solution operates as required.

6. [30 points]

Consider the program below written in the “Sample” language from Loudon (chap. 13). Show that the logical assertion $\{ \text{exp} * x^k = x^y \text{ and } k \geq 0 \}$ is an *invariant* of the loop appearing in the program (i.e., if the invariant and the loop guard are true before the execution of the loop body, then the invariant is still true after its execution).

```
pre-condition: {y > 0 }  
exp := 1; k := y;  
while k do  
  exp := exp * x;  
  k := k - 1  
od  
post-condition: { exp = xy }
```