We will treat the remaining operation specifications briefly, pausing only to take note of newly introduced features of Z.

RemoveEntry ________________________________

\[\begin{align*}
\text{PhoneDB} \\
\text{oldnumber?: Phone} \\
\text{name?: Person}
\end{align*}\]

\[\begin{align*}
\text{name?} & \mapsto \text{oldnumber?} \quad \square \quad \text{telephones} \\
\text{telephones} & ' = \text{telephones} \setminus \{\text{name?} \mapsto \text{oldnumber?}\} \\
\text{members'} & = \text{members}
\end{align*}\]

The pre-condition for removing an entry requires its presence, so we again have an exceptional condition to treat. The post-condition is expressed using a new pre-defined Z operation — set difference. **Set difference** denotes the set consisting of all elements in the first set but not in the second.

UnknownEntry _____________________________

\[\begin{align*}
\text{PhoneDB} \\
\text{oldnumber?: Phone} \\
\text{name?: Person} \\
\text{rep!: Report}
\end{align*}\]

\[\begin{align*}
\text{name?} & \mapsto \text{oldnumber?} \quad \square \quad \text{telephones} \\
\text{rep!} & = '\text{Unknown entry}'
\end{align*}\]

Then the ultimate operation specification is given as usual.

\[\text{DoRemoveEntry} \triangleq \text{RemoveEntry} \sqcap \text{Success} \sqcap \text{UnknownEntry}\]

The comparison with the Miranda animation again reveals a close match.
addMember n (mem, tel) = (mem ++ [n], tel)

doAddMember (mem, tel) n = write "Okay\n" (phdb (addMember n (mem, tel)))).if ~member mem n
= write "Already member\n" (phdb (mem, tel)), otherwise

The next PhoneDB operation brings us to other new pre-defined operations in Z.

The first pertains to a relation $\mathcal{R} \subseteq X \times Y$. Given a subset $W \subseteq X$, the **domain restriction** of $\mathcal{R}$ to $W$ is the relation $W \leftarrow \mathcal{R}$ defined by

$$W \leftarrow \mathcal{R} = \{(x,y) | x \in W \land (x,y) \in \mathcal{R}\}.$$ 

This permits the identification of the subdomain $W$ of the relation to be included.

The second pre-defined Z operation needed here is **anti-restriction** (or domain subtraction). It also pertains to a relation $\mathcal{R} \subseteq X \times Y$. Given a subset $W \subseteq X$, the anti-restriction of $\mathcal{R}$ to $W$ is the relation $W \leftarrow R$ defined by

$$W \leftarrow R = \{(x,y) | x \in W \land (x,y) \in \mathcal{R}\}.$$ 

The Miranda animation code directly follows this specification too.
This permits the identification of a subdomain $W$ of the relation to be *excluded*. Domain subtraction is used in the last operation specification.

```
RemoveMember ________________
[]PhoneDB
name?: Person

______________
name? [] members
members' = members \ {name?}
telephones' = {name?} \ telephones

And finally,
DoRemoveMember \= RemoveMember [] Success
    \= NotMember
```

This completes the specification of the PhoneDB operations, and its Miranda counterpart is also routine.

```
removeMember n (mem, tel) = (mem -- [n], ndres [n] tel)
doRemoveMember (mem, tel) n \ correction added
    = write "Okay\n" (phdb (removeMember n (mem, tel))), if member mem n
    = write "Not a member\n" (phdb (mem, tel)), otherwise

ndres u f = [(x,y) \f (x,y) <-f; ~member u x]
```