We have completed the specification of the first phone database operation, and now continue with those remaining. For the next operation we need to introduce a new formalism. This consists of using a relation to perform mappings as we normally do with functions. For a relation \( R \subseteq X \times Y \), each \( W \subseteq X \) is associated with its relational image \( R(W) = \{ y \in Y \mid \exists x \in W \quad xRy \} \).

The next PhoneDB operation is one of the “lookup” operations.

\[
\text{FindPhones} \quad \text{----------------------------------------} \\
\quad \text{PhoneDB} \\
\quad \text{name?: Person} \\
\quad \text{numbers!: \text{\textit{P}} Phone} \\
\quad \quad \text{name?: dom telephones} \\
\quad \quad \text{numbers! = telephones(\{name?\})} \\
\]

The relational image operation allows us to establish the desired post-condition directly, and the required pre-condition is evident. When the pre-condition is not met, the exceptional outcome needs to be explicitly stated. This is handled similarly to the previous case.

\[
\text{UnknownName} \quad \text{----------------} \\
\quad \text{PhoneDB} \\
\quad \text{name?: Person} \\
\quad \text{rep!: Report} \\
\quad \quad \text{name? \subseteq dom telephones} \\
\quad \quad \text{rep! = 'Unknown name'} \\
\]

The pre-condition here is just the negation of that for the FindPhones operation, and the post-condition indicates the error report. Then we again use a schema-formula to define

\[ \text{DoFindPhones} \triangleq \text{FindPhones} \triangleright \text{Success} \]

\[ \text{UnknownName} \]

If we pause to examine the corresponding component of the Miranda animation, we find a clear reflection of the specification.

\[
\text{findPhones n (mem, tel) = disp (image tel [n])} \\
\text{doFindPhones (mem, tel) n \quad \text{II correction added}} \\
\quad = \text{write (findPhones n (mem, tel) ++ "\n\n") (phdb (mem, tel))}, \\
\quad \text{if member (domain tel) n} \\
\quad = \text{write "Unknown name\n" (phdb (mem, tel))}, \text{ otherwise} \\
\]

\[
\]
image f u = [y | (x,y) <- f; member u x]

disp x = "Empty\n", if x = [ ]
  = hd x, if #x = 1
  = hd x ++ "\n" ++ disp (tl x), otherwise

We continue with the operation for looking up names. With the state space adopted, this leads us to the use of the relational inverse (or transpose) operation.

FindNames  
\[\text{FindNames} \quad \text{------------------------} \]
\[\text{\textbullet PhoneDB} \]
\[\text{names!} : \text{P Person} \]
\[\text{number?} : \text{Phone} \]

\[\text{number} \in \text{ran telephones} \]
\[\text{names!} = \text{telephones} \sim \{(\text{number})\} \]

The notation in Z for relational inverse is the postfix operator \(\sim\). For relation \(R \subseteq X \times Y\) and each \(W \subseteq Y\), this is defined as \(R \sim (W) = \{ x | xRy \text{ and } y \in W \}\). The precondition for FindNames insures that the names! result will be a non-empty set.

As was done with the FindEntry operation, we complete the specification by describing error handling.

UnknownNumber  
\[\text{UnknownNumber} \quad \text{------------------------} \]
\[\text{\textbullet PhoneDB} \]
\[\text{number?} : \text{Phone} \]
\[\text{rep!} : \text{Report} \]

\[\text{number} \in \text{ran telephones} \]
\[\text{rep!} = \text{'Unknown number'} \]

Then the completed specification is

\[\text{DoFindNames} \triangleq \text{FindNames} \uplus \text{Success} \]
\[\text{UnknownNumber} \]

Again the match with the Miranda animation should be clear.

\[\text{findNames } e (\text{mem, tel}) = \text{disp} (\text{image} (\text{inverse} \text{ tel}) [e]) \]
\[\text{doFindNames } (\text{mem, tel}) e \quad \text{II correction added} \]
\[= \text{write} (\text{findNames } e (\text{mem, tel}) ++ \text{"\n\n") \text{(phdb (mem, tel)),}}\]
if member (range tel) e
    = write "Unknown extension\n" (phdb (mem,tel)), otherwise

range f = [y | (x,y) <- f]

inverse f = [(y,x) | (x,y) <- f]