# CONFERENCE ON GEOMETRIC METHODS IN REPRESENTATION THEORY

## UNIVERSITY OF IOWA – IOWA CITY
## NOVEMBER 17-19, 2018

Conference Schedule

All talks are in MACLEAN HALL, rooms MacLean 110 and MacLean 118.

| | **Saturday, November 17** |
|---|---|
| 8:45-9:00 | WELCOME (118 MLH) |
| 9:00-10:00 | **Yakimov** (keynote) (118 MLH)<br>*Irreducible representations of Sklyanin algebras.* |
| 10:00-10:10<br>10:10-10:40 | Questions<br>Coffee |
| 10:40-11:30 | **Chinburg** (expository) (118 MLH)<br>*Group Cohomology Confidential: Sharing secrets with cup products.* |
| 11:30-11:45 | Questions |
| 11:45-12:15 | **Derksen** (118 MLH)<br>*Upper and lower degree bounds for generating invariants.* |
| 12:15-12:25 | Questions |
| 12:25-2:30 | LUNCH BREAK |
| 2:30-3:00 | **Webb** (118 MLH)<br>*Sets with an action of a category.* |
| 3:00-3:10<br>3:10-3:40 | Questions<br>Coffee and move to parallel talks |
| 3:40-4:00 | **Rajchgot** (118 MLH) *Type D quiver rep.varieties, double Grassmannians, and symm.varieties.*<br>**Serhiyenko** (110 MLH) *Cluster structures in Schubert varieties.* |
| 4:00-4:10 | Questions |
| 4:10-4:30 | **Allman** (118 MLH) *Quantum dilogarithm identities: the geometry of quiver rep.s viewpoint.*<br>**Meyer** (110 MLH) *Algebraic stability for arbitrary orientations of $\mathbb{A}_n$.* |
| 4:30-4:40 | Questions |

| | **Sunday, November 18** |
|---|---|
| 9:00-10:00 | **Yakimov** (keynote) (118 MLH)<br>*Quantum symmetric pairs.* |
| 10:00-10:10<br>10:10-10:40 | Questions<br>Coffee |
| 10:40-11:30 | **Lin** (expository) (118 MLH)<br>*Smooth representations of graded algebras.* |
| 11:30-11:45 | Questions |
| 11:45-12:15 | **Carlson** (118 MLH)<br>*Nearly null maps, virtual projectivity and generation of the stable category.* |
| 12:15-12:25 | Questions |
| 12:25-2:30 | LUNCH BREAK |
| 2:30-3:00 | **Seceleanu** (118 MLH)<br>*Reflection groups and the geometry of polynomial interpolation.* |
| 3:00-3:10<br>3:10-3:30 | Questions<br>Coffee |
| 3:30-4:00 | **Weyman** (118 MLH)<br>*Green Conjecture for general canonical curves.* |
| 4:00-4:10 | Questions |

| | **Monday, November 19** |
|---|---|
| 9:00-9:30 | **Drupieski** (118 MLH)<br>*Support schemes for infinitesimal unipotent supergroups.* |
| 9:30-9:40<br>9:40-10:10 | Questions<br>Coffee and move to parallel talks |
| 10:10-10:30 | **Matherne** (118 MLH) *Singular Hodge theory of matroids.*<br>**Makam** (110 MLH) *Weyl's polarization theorem in positive characteristic.* |
| 10:30:10:40 | Questions |
| 10:40-11:00 | **Iusenko** (118 MLH) *Schofield's theorem for poset representations.*<br>**Kulkarni** (110 MLH) *A combinatorial Fourier transform for quiver rep. varieties in type A.* |
| 11:00-11:10 | Questions |
| 11:10-11:30 | **Lőrincz** (118 MLH) *Representation varieties of algebras with nodes.*<br>**Sistko** (110 MLH) *Finiteness properties for automorphism classes of maximal subalgebras.* |
| 11:30-11:40 | Questions |
| 11:40-12:00 | **Gandini** (118 MLH) $\mathrm{GL}(\mathbf{V})$*-equivariant ideals associated to subspace arrangements.*<br>**Collins** (110 MLH) *Generalized Littlewood-Richardson coefficients for branching rules of* $\mathrm{GL}(n)$*.* |
| 12:00-12:10 | Questions |

## Abstracts

### Keynote Lectures

**Milen Yakimov (Louisiana State University).**

Talk 1: *Irreducible representations of Sklyanin algebras.*

Noncommutative projective schemes are abelian categories associated to N-graded connected algebras. Among those, the projective spaces come from Artin-Schelter regular algebras. The main examples of the latter in low dimensions are the Sklyanin algebras associated to pairs of an elliptic curve and an automorphism. We will make a brief overview of this subject and present a classification of the irreps of the 3 and 4 dim Sklyanin algebras for finite order automorphisms of elliptic curves ("modular case"). This is a joint work with Chelsea Walton and Xingting Wang.

Talk 2: *Quantum symmetric pairs.*

In 1999 Gail Letzter gave a general construction of symmetric pairs for quantized enveloping algebras. We will make an overview of the subsequent developments and explain a recent work with Stefan Kolb which describes the symmetric pairs admitting Iwasawa decomposition in the framework of diagonal Nichols algebras. This more general setting not only gives new examples like quantum groups at roots of unity, but also provides a more conceptual approach to the construction of quantum symmetric pairs. Based on this and a notion of star-products on N-graded connected algebras, it will be proved that all such algebras give rise to representations of the braid groups of type B.

**Ted Chinburg (University of Pennsylvania).**

*Group Cohomology Confidential: Sharing secrets with cup products.*

In this talk I will describe some potential connections between a problem in cryptography, the cohomology of groups and arithmetic geometry. The origin of the problem was to devise a secure system for a group of people to share a common secret based only on information they post to a public bulletin board. Boneh and Silverberg formulated this in terms of what they call cryptographic multilinear maps on finite dimensional vector spaces over a finite field. The construction of such multilinear maps involving more than two arguments has been an open problem for 15 years. I will describe some efforts in this direction using cup products coming from group theory and etale cohomology over varieties over finite fields. One goal of the talk is to make the underlying problem accessible to experts in representation theory who may think of new constructions.

**Zongzhu Lin (Kansas State University).**

*Smooth representations of graded algebras.*

A $\mathbb{Z}$-graded vector algebra $A = \oplus_n A_n$ has a natural filtration $F^\bullet A$ with

$$(F^p A)_n = \sum_{i \leq p} A_{n-i} A_i$$

which defines a Hausdorff linear topology on $A$. Then we consider the completion as well as the smooth representations of $A$ with respect to this topology (in the sense of representations of $p$-adic groups). One of the motivating examples is the construction of the universal enveloping algebras of a vertex algebra where the operator product in general are not defined in algebraic sense, but can make sense in the topological sense. The smooth representations of the universal enveloping algebras of vertex algebras are exactly the weak modules vertex algebras. Using this construction, the induction functors of vertex algebras can be easily constructed with tools available from representations of algebras.

## Conference Talks

**Justin Allman (U.S. Naval Academy).**
*Quantum dilogarithm identities: the geometry of quiver representations viewpoint.*
Quantum dilogarithm identities for quivers are important in the study of Donaldson-Thomas invariants, Poincaré series in cohomological Hall algebras of quivers, maximal green sequences in cluster algebra theory, counting BPS states in physics, and partition counting in combinatorics. In this talk we give an overview of the algebraic geometry/topology viewpoint on these identities, and some recent progress in this vein.

**Jon Carlson (University of Georgia).**
*Nearly null maps, virtual projectivity and generation of the stable category.*
This is joint work with Dave Benson. We consider maps that induce the zero map in high degrees of cohomology. The analogous idea for objects is virtual relative projectivity, high degree cohomology of one object factoring through a tensor product with another. These notions have some interesting connections with generation of the stable category for the modular group algebra of a finite group as well as for any supergroup scheme.

**Brett Collins (Fitchburg State University).**
*Generalized Littlewood-Richardson coefficients for branching rules of* $\mathrm{GL}(n)$.
Littlewood-Richardson coefficients are fundamental structure constants, yet prove to be extremely difficult to calculate in general. Following the methods of Derksen-Weyman and Chindris, I will describe how certain sums of products of Littlewood-Richardson coefficients, specifically the ones appearing as the multiplicities in the branching rules of $\mathrm{GL}(n)$, can be described as the dimensions of weight spaces of semi-invariants for certain quivers. I'll then discuss the consequences of this quiver theoretic description, including their saturation, a recursive procedure for finding all which are nonzero, and how determining whether or not they're nonzero can be done in polynomial time.

**Harm Derksen (University of Michigan).**
*Upper and lower degree bounds for generating invariants.*
I will give examples of families of representations for which there exists a polynomial bound for the degrees of generators of the invariant ring. I will also present exponential lower bounds of Visu Makam and the speaker for generating invariants of the SL(n)-action on 4-tuples of cubic forms.

**Christopher Drupieski (DePaul University).**
*Support schemes for infinitesimal unipotent supergroups.*
To each modular representation $M$ of a finite group (scheme) $G$, one can associate a corresponding geometric invariant $|G|_M$ called the (cohomological) support variety of $M$. By studying the structure of these geometric invariants, one can often gain new insights into the representation theory of $G$. However, support varieties are difficult to calculate explicitly because the cohomology rings used to define them are often difficult to calculate explicitly.

Thus, a first step in studying support varieties is to find some sort of non-cohomological representation-theoretic description for the variety $|G|_M$, which will hopefully be easier to compute in practice. In this talk I will discuss recent work with Jonathan Kujuwa on support varieties for infinitesimal unipotent supergroup schemes, in which we prove a "rank variety" type description for the varieties in question. (This builds on the work discussed by Julia Pevtsova at CGMRT V in 2017.)

**Francesca Gandini (University of Michigan).**
$\mathrm{GL}(\mathbf{V})$-*equivariant ideals associated to subspace arrangements.*

Suppose that $W_1, W_2, \ldots, W_t$ are subspaces of an $m$-dimensional $\mathbb{K}$-vector space $W \cong \mathbb{K}^m$ and let $J_1, J_2, \ldots, J_t \subseteq \mathbb{K}[x_1, x_2, \ldots, x_m]$ be the vanishing ideals of $W_1, W_2, \ldots, W_t$. Taking the tensor product of $W$ with a $n$-dimensional vector space $V$, we obtain $\mathrm{GL}(V)$-equivariant ideals in the polynomial ring in $mn$ variables. In particular, we define $J_i(V)$ to be the vanishing ideal of $W_i \otimes V$ for $i = 1, \ldots, t$. The product of these ideals $J_1(V)J_2(V)\cdots J_t(V)$ is also $\mathrm{GL}(V)$-equivariant and we can view it as an ideal in the symmetric algebra $S(W \otimes V)$. We can study the product ideal by considering the polynomial functor sending a vector space $V$ to the $\mathrm{GL}(V)$-representation $J_1(V)J_2(V)\cdots J_t(V)$. On the other hand, we can also take the wedge product of these ideals to get a $\mathrm{GL}(V)$-equivariant ideal in the exterior algebra $\bigwedge(W \otimes V)$. We also have a polynomial functor sending a vector space $V$ to the $\mathrm{GL}(V)$-representation $J_1(V) \wedge J_2(V) \wedge \cdots \wedge J_t(V)$. We rely on a functor $\Omega$ from the category of polynomial functors to itself to transfer information from the product ideal to the wedge ideal of a subspace arrangement.

**Kostiantyn Iusenko (University of Sao Paulo).**
*Schofield's theorem for poset representations.*

In 1991 Aidan Schofield gave a characterization of Schurian roots for the quiver $Q$. This characterization is given in terms of the bilinear form $\langle \cdot, \cdot \rangle$ associated with $Q$. Namely, given dimension vector $\alpha$ is a Schur root if and only if for a general representation $X$ of $Q$ with dimension $\alpha$ we have that $\langle \beta, \alpha \rangle - \langle \alpha, \beta \rangle > 0$ for all proper subrepresentations of $X$ with dimension $\beta$. This characterization turned out to be of great importance for stable representations of quiver. I plan to discuss a direct generalization of Schofield's characterization to bound quivers which appears considering representations of partially ordered sets.

**Maitreyee Kulkarni (Louisiana State University).**
*A combinatorial Fourier transform for quiver representation varieties in type A.*

For a given dimension vector d, we consider the space of representations of the linearly-oriented type A quiver. A product of general linear groups acts on this space, and the orbits are isomorphism classes of representations with dimension vector d. In this setting, we introduce a combinatorial algorithm to describe the Fourier–Sato transform; this algorithm matches up orbits for the type A quiver with orbits for its reversed quiver in an interesting way. The combinatorial Fourier–Sato algorithm and its inverse both give the same map as the Knight–Zelevinsky multisegment duality. The only proof we know that these three algorithms are the same is purely geometric. This is joint work with Pramod Achar and Jacob Matherne.

**András Lőrincz (Purdue University).**
*Representation varieties of algebras with nodes.*

In this talk I discuss the behavior of representation varieties of quivers with relations under the operation of node splitting. Working in the relative setting (splitting one node at a time) allows us to combinatorially enumerate irreducible components of representation varieties, and show they have rational singularities, for a wide class of algebras. This class contains all radical square zero algebras but also many others. We also give applications to generic decomposition within irreducible components and decomposition of moduli spaces of semistable representations of certain algebras. This is joint work with Ryan Kinser.

**Visu Makam (IAS Princeton).**
*Weyl's polarization theorem in positive characteristic.*

Let $V$ be a rational representation of a reductive group $G$ over an algebraically closed field $K$. For a positive integer $m$, consider the invariant rings $K[V^m]^G$ for the action of $G$ on a direct sum of $m$-copies of $V$. In characteristic zero, there is a stabilization result due to Weyl. More precisely, it says that if $S$ is a generating set for $K[V^n]^G$, then one can obtain a generating set for $K[V^m]^G$ using polarization no matter how large $m$ is. In positive characteristic, this fails in general. However, when $V$ is a good $G$-module, we show that the statement of Weyl's theorem holds for sufficiently large characteristic. To obtain our results, we combine ideas from representation theory in positive characteristic, combinatorics of Young tableau, commutative algebra and algebraic geometry. This is joint work with Harm Derksen.

**Jacob Matherne (University of Massachusetts Amherst).**
*Singular Hodge theory of matroids.*

To any matroid, I will associate a certain ring that, when the matroid is realizable, is the cohomology ring of a certain variety called the semi-wonderful model. I will show how the Hodge theory of this ring can conjecturally be used to establish the "top-heavy conjecture" of Dowling and Wilson from 1974, as well as the non-negativity of the Kazhdan-Lusztig polynomials of Elias, Proudfoot, and Wakefield. This is in progress joint work with Tom Braden, June Huh, Nicholas Proudfoot, and Botong Wang.

**David Meyer (Smith College).**
*Algebraic stability for arbitrary orientations of $\mathbb{A}_n$.*

Persistent homology uses persistence modules to try to distinguish the legitimate topological features of a data set from noise. These persistence modules are nothing more than representations of posets. Often, the collection of persistence modules is endowed with several metric structures. An algebraic stability theorem compares them, typically showing that the identity is an isometry or a contraction. Persistence modules that come from data correspond to representations of the equioriented $\mathbb{A}_n$ quiver. Since any orientation of $\mathbb{A}_n$ is the Hasse quiver of a finite poset, it's natural to wonder whether one can prove such a stability theorem for these posets. We compare various metrics on persistence modules in this setting.

**Jenna Rajchgot (University of Saskatchewan).**
*Type D quiver representation varieties, double Grassmannians, and symmetric varieties.*

Consider the following three families of varieties:

(1) GL-orbit closures in type $D$ quiver representation varieties, where GL is the base change group acting by conjugation,
(2) $B$-orbit closures in double Grassmannians $G/P_1 \times G/P_2$, where $B \leq G$ is a Borel subgroup acting diagonally,
(3) $B$-orbit closures in symmetric varieties $G/K$, where $G = \mathrm{GL}(p+q)$ and $K = \mathrm{GL}(p) \times \mathrm{GL}(q)$.

After recalling some background, I will explain how certain questions about geometry and combinatorics of the varieties in one of these families can be studied by considering the corresponding questions for varieties in either of the other two families. This is joint work with Ryan Kinser.

**Alexandra Seceleanu (University of Nebraska-Lincoln).**
*Reflection groups and the geometry of polynomial interpolation.*

The interpolation problem in algebraic geometry asks for the equations of polynomials in several variables passing through a given set of points in the plane with assigned multiplicities. There are many beautiful results and long-standing conjectures regarding the dimensions of the linear spaces formed by the interpolation polynomials and the degrees of these polynomials. We consider the implications of symmetry on the interpolation problem through the lens of several examples where the interpolation points arise from the action of a reflection group on the complex plane. We use classical invariant theory to show that the ideal of these interpolation points has a surprising property from the point of view of commutative algebra. This is based on joint work with Thomas Bauer, Sandra Di Rocco, Brian Harbourne, Jack Huizenga, and Tomasz Szemberg and also on work of Benjamin Drabkin.

**Khrystyna Serhiyenko (University of California, Berkeley).**
*Cluster structures in Schubert varieties.*

Coordinate rings of many important variates from Lie theory have cluster algebra structure. In this talk, we will discuss cluster structures in Schubert varieties of the Grassmannian and their categorification via representation theory of preprojective algebras. In particular we relate combinatorics of Postnikov's plabic graphs and results of Leclerc, who constructs cluster subalgebras in coordinate rings of Richardson varieties in the complete flag variety. This combinatorial interpretation naturally generalizes the work of Scott for the Grassmannian, and has been conjectured for some time. This is joint work with M. Sherman-Bennett and L. Williams.

**Alex Sistko (University of Iowa).**
*Finiteness properties for automorphism classes of maximal subalgebras.*

Let $k$ be an algebraically-closed field, $Q$ a finite quiver, and $B = kQ/I$ a bound quiver algebra with $n = \dim_k B < \infty$. Recently, it was shown that the projective variety of all $(n-1)$-dimensional subalgebras of $B$ only depends on the underlying quiver $Q$, and can be thought of as the variety of all maximal subalgebras of $B$. We call this variety $\mathfrak{msa}(Q)$. In this talk, we review the basic structure theory of $\mathfrak{msa}(Q)$, and discuss the problem of determining the admissible ideals $I$ for which $\mathfrak{msa}(Q)$ is a finite union of $\mathrm{Aut}_k(kQ/I)$-orbits. Using structure theorems for maximal subalgebras, we show that finiteness holds whenever $Q$ is Schur or $I$ is a monomial ideal. We also introduce a class of ideals $I$, closely related to quiver representations, for which the following hold: $Q$ is generally not Schur, and $I$ is generally not monomial, but $\mathfrak{msa}(Q)$ is still a finite union of $\mathrm{Aut}_k(kQ/I)$-orbits.

**Peter Webb (University of Minnesota).**

*Sets with an action of a category.*

Within group theory, permutation representations have always played a key role in understanding just about every aspect of a group. More recently bisets (sets with commuting actions of a pair of groups), as well as representations of a category where bisets are morphisms, have been studied as providing an extension of the theory of Mackey functors on groups. The construction of the Burnside ring of a finite group provides a projective object among representations of this biset category. These things can be done with arbitrary small categories, not just with groups. The bisets have been studied for a long time and are known as profunctors, or distributors. We consider two aspects of this extension of the theory: what we should mean by the Burnside ring of a category, and properties of biset functors for categories.

**Jerzy Weyman (University of Connecticut).**

*Green Conjecture for general canonical curves.*

Green conjecture predicts the shape of free resolution of the coordinate ring of curve of genus g embedded by canonical divisor. In the case of general curve of genus g the conjecture was proved by Claire Voisin in two breakthrough papers about fifteen years ago. I will discuss a new proof of this result, based on joint work with Marian Aprodu, Gabi Farkas, Stefan Papadima and Claudiu Raicu.