Derived Tame Nakayama Algebras

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Set Up

In this talk:

- $\Bbbk$ is an algebraically closed field of arbitrary characteristic.
- Λ denotes a fixed basic connected finite-dimensional $\Bbbk$-algebra.
- Λ-mod denotes the abelian category of finitely generated left Λ-modules, and Λ-proj denotes the subcategory of Λ-mod of projective objects.
- Unless explicitly stated, all modules are finitely generated and from the left side.
- $\mathcal{D}^b(\Lambda\text{-mod})$ denotes the bounded derived category of bounded complexes whose terms are in Λ-mod, and $\mathcal{K}^b(\Lambda\text{-proj})$ denotes the category of perfect complexes over Λ.

Definition 1. Let $C^\bullet$ be a complex in $\mathcal{D}^b(\Lambda\text{-mod})$. The **cohomology dimension** of $C^\bullet$ is the vector

$$\textbf{h-dim}\, C^\bullet = (\textbf{dim}\, \mathrm{H}^i(C^\bullet))_{i\in\mathbb{Z}},$$

where for all $i \in \mathbb{Z}$, $\textbf{dim}\, \mathrm{H}^i(C^\bullet)$ denotes the **dimension vector** of $\mathrm{H}^i(C^\bullet)$.

Definition 2 ((CH. GEISS, H. KRAUSE, 2002))**.** Λ is said to be **derived tame** if for every vector $\mathbf{n} = (n_i)_{i\in\mathbb{Z}}$ of natural numbers there exists a localization $R = \Bbbk[t]_f$ with respect to some $f \in \Bbbk[t]$ and a finite number of bounded complexes of R-Λ-bimodules $C_1^\bullet, \dots, C_k^\bullet$ such that each C_i^j is finitely generated free over R and (up to isomorphism) all but finitely many indecomposable objects of cohomology dimension $\mathbf{n}$ in $\mathcal{D}^b(\Lambda\text{-mod})$ are of the form $S \otimes_R C_i^\bullet$ for some $i \in \{1, \dots, k\}$ and some simple R-module S.

Theorem 3 ((CH. GEISS, H. KRAUSE, 2002))**.** *Derived tameness is preserved by derived equivalence.*

- Every derived discrete algebra is derived tame (D. VOSSIECK, 2001).
- If Λ has finite global dimension, then Λ is derived tame if and only if its repetitive algebra $\hat{\Lambda}$ is tame (CH. GEISS, H. KRAUSE, 2002).
- If Λ is piecewise hereditary, then Λ is derived tame (CH. GEISS, 2002).

Definition 4. Assume that Λ has finite global dimension. The **Euler form** χ_Λ of Λ is defined on the Grothendieck group of Λ by

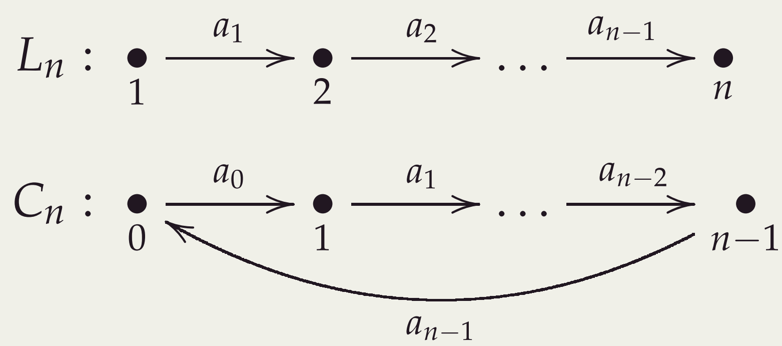
$$\chi_\Lambda(\mathbf{dim}\, M) = \sum_{i=0}^{\infty}(-1)^i \dim_{\Bbbk} \operatorname{Ext}^i_\Lambda(M, M)$$

for every Λ-module M.

- If Λ is a tree algebra, then Λ is derived tame if and only if χ_Λ is non-negative (TH. BRÜSTLE, 2001).
- If Λ is either gentle or skewed-gentle, then Λ is derived tame (V. BEKKERT, H. MERKLEN, 2003) AND (V. BEKKERT, E. N. MARCOS, H. MERKLEN, 2003).

Recall that Λ is said to be a **Nakayama algebra** if every left or right indecomposable projective Λ-module has a unique composition series.

Theorem 5. *Λ is a Nakayama algebra if and only if $\Lambda = \Bbbk Q/I$, where Q is one of the following quivers:*



for some $n \geq 1$.

Assume that $\Lambda = \Bbbk Q/I$ is a Nakayama algebra.

- If $Q = L_n$, then we say that Λ is a **line algebra**.
- If $Q = C_n$, then we say that Λ is a **cycle algebra**.

Theorem 6. (V. BEKKERT, H. GIRALDO, V-M, IN PROGRESS) *Assume that Λ is a Nakayama algebra. Then Λ is derived tame if and only if one of the following conditions holds:*

(i) *Λ is a line algebra whose Euler form is non-negative.*
(ii) *Λ is either gentle or derived equivalent to some skewed-gentle algebra.*

Definition 7. (E. ENOCHS, O. JENDA, 1995) A Λ-module V is said to be **Gorenstein-projective** if there exists an acyclic complex of projective Λ-modules

$$P^\bullet : \cdots \to P^{-2} \xrightarrow{\delta^{-2}} P^{-1} \xrightarrow{\delta^{-1}} P^0 \xrightarrow{\delta^0} P^1 \xrightarrow{\delta^1} P^2 \to \cdots$$

such that $\mathrm{Hom}_\Lambda(P^\bullet, \Lambda)$ is also acyclic and $V = \mathrm{coker}\,\delta^0$. We denote by Λ-Gproj the category of Gorenstein-projective Λ-modules that are finitely generated, and by Λ-$\underline{\text{Gproj}}$ its stable category.

Definition 8. The **singularity category** of Λ is defined to be the Verdier quotient $\mathcal{D}^b(\Lambda\text{-mod})/\mathcal{K}^b(\Lambda\text{-proj})$.

Since Gorensteinness is preserved by derived equivalence (see e.g. (A. BELIGIANNIS, 2005)), and since gentle and skewed-gentle algebras are Gorenstein (see e.g. (CH. GEISS & I. REITEN, 2005) and (X. CHEN & M. LU, 2017)), by using (R.O. BUCHWEITZ, UNPUBLISHED) we obtain the following result.

Corollary 9. *If Λ is a derived tame Nakayama algebra, then Λ is Gorenstein, and consequently if Λ is a cycle algebra, then $\mathcal{D}_{sg}(\Lambda\text{-mod}) = \Lambda\text{-}\underline{\text{Gproj}}$.*

By using the description of the singularity category of a gentle algebra in (M. KALCK, 2015), we obtain the following result.

Corollary 10. *Let $\Lambda = \Bbbk Q/I$ is a derived tame cycle algebra, and let $|R_\Lambda|$ the minimal number of relations defining I. If Λ has infinite global dimension, then there exists an equivalence of triangulated categories*

$$\mathcal{D}_{sg}(\Lambda\text{-mod}) \cong \mathcal{D}^b(\Bbbk\text{-mod})/[|R_\Lambda|],$$

where $\mathcal{D}^b(\Bbbk\text{-mod})/[|R_\Lambda|]$ denotes the **orbit category** *(in the sense of* (B. KELLER, 2005)*.*

The following results classifies the isomorphism class of versal deformation rings of Gorenstein-projective modules (in the sense of (F. M. Bleher, V-M, 2012)) over derived tame Nakayama algebras.

Corollary 11. *Let* Λ *be a derived tame Nakayama algebra, and let* V *be in* Λ*-Gproj. If* V *is indecomposable, then the versal deformation ring* $R(\Lambda, V)$ *of* V *is universal and isomorphic either to* $\Bbbk$ *or to* $\Bbbk[[t]]/(t^2)$.

¡GRACIAS POR SU ATENCIÓN!

THANKS FOR YOUR ATTENTION!

MERCI DE VOTRE ATTENTION!

OBRIGADO PELA SUA ATENÇÃO!

DANK FÜR IHRE AUFMERKSAMKEIT!