CONFERENCE ON GEOMETRIC METHODS IN REPRESENTATION THEORY

UNIVERSITY OF MISSOURI-COLUMBIA

NOVEMBER 19-21, 2016

Conference Schedule

All talks are in STRICKLAND HALL, rooms Strickland 105 and Strickland 114.

	Saturday **November 19**
8:30-8:45	Registration
8:45-9:00	WELCOME
9:00-10:00	**Derksen** (keynote) (STRICK 114) *Invariant theory for quivers.*
10:00-10:10	Questions
10:10-10:40	Coffee/Registration
10:40-11:30	**Huisgen-Zimmermann** (expository) (STRICK 114) *Closures in varieties of representations.*
11:30-11:45	Questions
11:45-12:15	**Carlson** (STRICK 114) *Nilpotence and generation in the stable module category.*
12:15-12:25	Questions
12:25-2:30	LUNCH BREAK
2:30-3:00	**Kirkman** (STRICK 114) *Hopf actions on AS regular algebras: Auslander's theorem.*
3:00-3:10	Questions
3:10-3:30	Coffee
3:30-4:00	**Iovanov** (STRICK 114) *A new framework for uniserial representations.*
4:00-4:15	Questions and move to parallel talks
4:15-4:35	**Vashaw** (STRICK 105) *Prime spectra of 2-categories.* **Allman** (STRICK 114) *Donaldson-Thomas invariants for A-type square product quivers.*
4:35-4:45	Questions
6:30-9:00	CONFERENCE DINNER at Chris McD's

	Sunday **November 20**
9:00-10:00	**Derksen** (keynote) (STRICK 114) *Invariant theory for quivers.*
10:00-10:10	Questions
10:10-10:40	Coffee/Registration
10:40-11:30	**Weyman** (expository) (STRICK 114) *Towards the geometric characterization of finite representation type and tameness.*
11:30-11:45	Questions
11:45-12:15	**Thomas** (STRICK 114) *τ-tilting theory for gentle algebras.*
12:15-12:25	Questions
12:25-2:30	LUNCH BREAK
2:30-3:00	**Lin** (STRICK 114) *The varieties of semi-conformal vectors of vertex operator algebras.*
3:00-3:10	Questions
3:10-3:40	Coffee and move to parallel talks
3:40-4:00	**Witt** (STRICK 105) *Finding Lyubeznik numbers.* **Schaefer** (STRICK 114) *On the structure of generalized symmetric spaces of* $\mathrm{SL}_n(\mathbb{F}_q)$.
4:00-4:10	Questions
4:10-4:30	**Carroll** (STRICK 105) *Canonical join representations for torsion classes.* **Soto** (STRICK 114) *Universal deformation rings: semidihedral and quaternion 2-groups.*
4:30-4:40	Questions

	Monday **November 21**
9:30-10:00	**Chinburg** (STRICK 114) *Holomorphic differentials and support varieties.*
10:00-10:10	Questions
10:10-10:40	Coffee and move to parallel talks
10:30-10:40	Questions
10:40-11:00	**Shipman** (STRICK 105) *Representation schemes for DG algebras.* **Paquette** (STRICK 114) *Isotropic Schur roots.*
11:00:11:10	Questions
11:10-11:30	**Makam** (STRICK 105) *Generating invariants in positive characteristic.* **Lorincz** (STRICK 114) *Free resolutions of orbit closures of Dynkin quivers.*
11:30-11:40	Questions
11:40-12:00	**Livesay** (STRICK 105) *Moduli spaces of meromorphic* GSp*-connections.* **Granger** (STRICK 114) *GIT-equivalence and semi-stable subcategories.*
12:00-12:10	Questions
12:10-12:30	**Meyer** (STRICK 105) *Finite subgroups of* $\mathrm{GL}_2(\mathbb{C})$ *and universal deformation rings.* **Kline** (STRICK 114) *A conjecture of Victor Kac.*
12:30-12:40	Questions

Abstracts

Keynote Lectures

Harm Derksen (University of Michigan).
Invariant theory for quivers.

Given a quiver Q, the isomorphism classes of representations of a given dimension vector α correspond to orbits of the group GL_α in the representation space $\mathrm{Rep}_\alpha(Q)$. It is therefore natural to study the ring $I(Q,\alpha)$ of GL_α-invariant polynomial functions on $\mathrm{Rep}_\alpha(Q)$ and the ring $SI(Q,\alpha)$ of SL_α-invariant polynomials (also called the ring of semi-invariants).

I will discuss general notions from (Geometric) Invariant Theory and show how they apply to quiver representations. We will give various descriptions of generators for rings of invariants and semi-invariants, and also give degree bounds. In particular, I will also explain a recent result with Visu Makam showing that the ring of $\mathrm{SL}_n \times \mathrm{SL}_n$-invariants of m-tuples of $n \times n$ matrices is generated by invariants of degree $\leq n^6$. This invariant ring is the ring of semi-invariants for the generalized Kronecker quiver. Several applications, such as noncommutative identity testing and estimating ranks of tensors, will be discussed.

Expository Lectures

Birge Huisgen-Zimmermann (UC Santa Barbara).
Closures in varieties of representations.

We will determine the closures of certain key subvarieties of the varieties parametrizing the modules with fixed dimension vector over a finite dimensional algebra. The findings will be applied to the problem of pinning down the irreducible components of the parametrizing varieties.

Jerzy Weyman (University of Connecticut).
Towards the geometric characterization of finite representation type and tameness.

I will discuss various possible characterizations of finite representation type and tameness in terms of geometry of representation spaces and Geometric Invariant Theory. The talk is based on the joint work with Andrew Carroll, Calin Chindris, Ryan Kinser and Amelie Schreiber.

Conference Talks

Justin Allman (U.S. Naval Academy).
Donaldson-Thomas invariants for A-type square product quivers.

The square product of two A-type quivers is a 2 dimensional grid of oriented cycles. For a fixed dimension vector, the number of indecomposable quiver representations is infinite, but in this talk we will describe a stratification of the representation space with finitely many strata. On each of these strata we count the codimension and a new combinatorial invariant, which we call the superpotential contribution, coming from computation of the Poincaré series for equivariant rapid-decay cohomology for quivers with potential (defined in the talk). This data can be encoded simultaneously for all dimension vectors in an identity of quantum dilogarithm series, giving a geometric and topological interpretation of Keller's combinatorial Donaldson-Thomas invariant.

Jon Carlson (University of Georgia).
Nilpotence and generation in the stable module category.

Nilpotence has been studied in stable homotopy theory and algebraic geometry. We study the corresponding notion in modular representation theory of finite groups, and apply the discussion to the study of ghosts, and generation of the stable module category. In particular, we show that for a finitely generated kG-module M, the tensor M-generation number and the tensor M-ghost number are both equal to the degree of tensor nilpotence of a certain map associated with M. This paper is joint work with Dave Benson.

Andy Carroll (DePaul University).
Canonical join representations for torsion classes.

The lattice structure of the poset of torsion classes over a finite-dimensional associative algebra has recently seen a resurgence of attention. Beautiful connections between these posets and the combinatorics of the weak order on associated Coxeter groups have been explored by Mizuno, and Iyama-Reading-Reiten-Thomas, the latter of which included a representation-theoretic description of the objects known as shards, which appear in the combinatorics of reflecting hyperplane arrangements for said Coxeter groups. In this talk, I will review some of these connections, and describe new work that puts these results in a more general context. In particular, we give representation-theoretic interpretations of the join-irreducible torsion classes, and the so-called canonical join representations of elements. This talk is based on joint work with Emily Barnard, Gordana Todorov, and Shijie Zhu.

Ted Chinburg (University of Pennsylvania).
Holomorphic differentials and support varieties.

It is a classical problem to describe the action of a finite group G on the holomorphic differentials $H^0(X, \Omega^1_{X/k})$ of a smooth projective curve X over a field k on which G acts over k. This problem is much more subtle when k has characteristic $p > 0$ and p divides the order of G. I will discuss a proof of the fact that if G is an elementary abelian p-group, the non-maximal support variety of $H^0(X, \Omega^1_{X/k})$ is contained in a union of projective spaces associated to the inertia subgroups of the action of G on X.

Valerie Granger (Coker College).
GIT-equivalence and semi-stable subcategories of quiver representations.

Quivers are simply directed graphs. Quivers and their representations occur most naturally in representation theory of finite dimensional algebras. In the context of quiver invariant theory, the main objects of study are the so-called semi-stable representations. These are representations that obey certain linear homogeneous inequalities. I will investigate the question of when two semi-stability conditions give rise to the same semi-stable subcategory, in which case we say they are GIT-equivalent. This work extends previous work of Colin Ingalls, Charles Paquette, and Hugh Thomas to the wild case. The necessary and sufficient condition for GIT-equivalence discussed is given by a collection of cones parametrized by Schur roots. In particular, this description recovers the results of Ingalls, Paquette, and Thomas in the case that the quiver is tame. This is joint work with Calin Chindris.

Miodrag Iovanov (University of Iowa).
A new framework for uniserial representations.

We present a new framework for studying uniserial representations of finite dimensional algebras. This is based on rational actions of the path algebra on the path coalgebra, which are used to find canonical combinatorial bases of such modules. This work in progress is motivated by the questions on uniserial modules that appeared the textbook of Auslander, Reiten, and Smalo, and work of B. Huisgen-Zimmermann and K. Bongartz who solved to two of these problems. Time permitting, we will explain how some new solutions to such problems can be obtained in our set-up.

Ellen Kirkman (Wake Forest University).
Hopf actions on AS regular algebras: Auslander's theorem.

Let $\Bbbk$ be an algebraically closed field of characteristic zero. Maurice Auslander proved that when a finite subgroup G of $\mathrm{GL}_n(\Bbbk)$, containing no reflections, acts on $A = \Bbbk[x_1, \ldots, x_n]$ naturally, with fixed subring A^G, then the skew group algebra $A\#G$ is isomorphic to $\mathrm{End}_{A^G}(A)$ as algebras. In work with K. Chan, C. Walton and J.J. Zhang, we prove that $A\#H$ is isomorphic to $\mathrm{End}_{A^H}(A)$ as algebras when A is an Artin-Schelter regular algebra of dimension 2 and H is a semisimple Hopf algebra, acting on A so that A is a graded H-module algebra under an action that is inner faithful and has trivial homological determinant; in this case A^H is a graded isolated singularity in the sense of Mori-Ueyama. With Gaddis, Moore, and Won we prove that Auslander's Theorem holds for $A = \Bbbk_{-1}[x_1, \ldots, x_n]$ under the action of any group of permutations of $\{x_1, \ldots, x_n\}$.

Dan Kline (Milligan College).
A conjecture of Victor Kac.

In this talk, we solve a conjecture raised by Victor Kac in 1982 concerning the so called locally semi-simple representations of quivers. Our main result shows that an acyclic quiver is tame (i.e. either Dynkin or Euclidean) if and only if every representation with a semi-simple endomorphism ring is in fact locally semi-simple. This result confirms Kac's conjecture and shows the validity of Kac's conjecture precisely captures the tameness of a quiver.

Zongzhu Lin (Kansas State University).
The varieties of semi-conformal vectors of vertex operator algebras.

A vertex operator algebra is a vertex algebra equipped with a conformal structure, i.e., a module structure for the Virasoro Lie algebra determined by a conformal vector through state-field correspondence. Given a vertex algebra, there can be many different conformal structures. This talk is concerned about the moduli space structures of the conformal structures as well as the moduli spaces of the vertex operator subalgebras, which may not be a submodule of the Virasoro Lie algebra. One class of vertex operator subalgebra is called semi-conformal subalgebras which have been studied in many contexts such as coset constructions. In the Heisenberg vertex algebra cases, the geometric structure of all semi-conformal vectors completely determines the vertex operator algebra.

Neal Livesay (Louisiana State University).
Moduli spaces of meromorphic GSp*-connections.*

A fundamental problem in the theory of differential equations is the classification of first-order singular linear differential operators up to gauge equivalence. An algebra-geometric version of this problem involves the study of moduli spaces of meromorphic G-connections on $\mathbb{P}^1$ (where G is the gauge group) with specified local isomorphism classes. Recently, C. Bremer and D. Sage, inspired primarily by a 2002 paper of P. Boalch, developed a new method for the analysis of the local isomorphism classes using representation theoretic techniques. Furthermore, they have realized explicit symplectic moduli spaces for GL_n-connections where the local isomorphism classes satisfy certain regularity conditions. In this talk, I will describe some of the basic ideas involved in this project, discuss current work towards constructing moduli spaces of GSp_{2n}-connections, and demonstrate some low rank examples.

Andras Lorincz (Purdue University).
Free resolutions of orbit closures of Dynkin quivers.

In this talk, we show how one can construct the minimal free resolutions of 1-step orbit closures of Dynkin quivers. These can be viewed as generalizations of Lascoux's resolution for determinantal ideals. We use the resolutions to prove that such orbit closures are normal, Cohen-Macaulay and have rational singularities. This is joint work with Jerzy Weyman.

Visu Makam (University of Michigan).
Generating invariants in positive characteristic.

The ring of invariants for a rational representation of a reductive group is finitely generated by the results of Hilbert, Nagata and Haboush. However, the proof is not constructive in nature. While finding a minimal set of generators remains a difficult question, one could ask instead for an upper bound on the degrees of generators. The best known bounds in characteristic zero are due to Derksen, and it remains an open question in positive characteristic. We outline a strategy to compute bounds in positive characteristic when the coordinate ring has a good filtration. Using this strategy, we are able to obtain strong bounds for invariant rings associated to quivers for arbitrary characteristic. This is joint work with Harm Derksen.

David Meyer (University of Missouri-Columbia).
Finite subgroups of $\mathrm{GL}_2(\mathbb{C})$ *and universal deformation rings.*

Universal deformation rings convey information about the characteristic 0 representations associated to characteristic p representations of a group. Let Γ be a finite group, and let V be an absolutely irreducible $\mathbb{F}_p\Gamma$-module. We consider the function which assigns to V its universal deformation ring $R(\Gamma, V)$. When this function is nonconstant, we can use its graph to determine information about the internal structure of the group Γ. Specifically, we connect the fusion of certain subgroups N of Γ, to the kernels of those representations whose corresponding modules are a level set of the function $V \to R(\Gamma, V)$. We consider groups Γ which are extensions of finite irreducible subgroups of $\mathrm{GL}_2(\mathbb{C})$ by elementary abelian p-groups of rank 2.

Charles Paquette (University of Connecticut).
Isotropic Schur roots.

Let Q be an acyclic quiver and let k be an algebraically closed field. A representation of Q is *Schur* if it has a trivial endomorphism algebra, and a *Schur root* is the dimension vector of such a representation. A Schur root is *isotropic* if it is a zero of the Tits form of Q. In this talk, we give a description of the perpendicular category of any isotropic Schur root and describe its (infinitely many) non-isomorphic simple objects. As a consequence, we get a structural result for the ring of semi-invariants of any isotropic Schur root. We will also see how to construct all isotropic Schur roots of a given quiver Q, using the action of some braid group. This is joint work with Jerzy Weyman.

Jennifer Schaefer (Dickinson College).
On the structure of generalized symmetric spaces of $\mathrm{SL}_n(\mathbb{F}_q)$.

In this talk, we extend previous results regarding $\mathrm{SL}_2(k)$ over any finite field k by investigating the structure of the symmetric spaces for the family of special linear groups $\mathrm{SL}_n(k)$ for any integer $n > 2$. Specifically, we discuss the generalized and extended symmetric spaces of $\mathrm{SL}_n(k)$ for all conjugacy classes of involutions over a finite field of odd or even characteristic. Characterizations of the structure of these spaces and an explicit difference set will be provided in select cases where the two spaces are not equal. This joint work with C. Buell, A. Helminck, V. Klima, C. Wright, and E. Ziliak.

Ian Shipman (University of Utah).
Representation schemes for DG algebras.

Given a DG algebra, one can ask for a parameter space of DG modules over it. It turns out that there are several approaches to this question. I will discuss work in progress on both classical and derived approaches. The classical approach concerns parameterizing DG modules up to quasi-isomorphism with a scheme. The derived approach concerns the derived representation scheme of Y. Berest et al.

Roberto Soto (CSU Fullerton).
Universal deformation rings: semidihedral and generalized quaternion 2-groups.

Let k be an algebraically closed field of characteristic 2 and let W be the ring of infinite Witt vectors over k. Moreover, suppose D is a 2-group, either semidihedral of order at least 16 or (generalized) quaternion of order at least 8. In this talk we show that the universal deformation ring, $R(D, V)$, of every endo-trivial $k[D]$-module V is isomorphic to $W[\mathbb{Z}/2 \times \mathbb{Z}/2]$. This is joint work with F. Bleher and T. Chinburg.

Hugh Thomas (Université du Québec Montréal).
τ-tilting theory for gentle algebras.

In order to get a good combinatorial description of τ-tilting theory for gentle algebras, one needs a description of when $\mathrm{Hom}(X, \tau Y)=0$ for X and Y indecomposable τ-rigid modules. In principle this can be determined, in that morphisms between string modules and AR-translation for gentle algebras are both well-understood; there has also been recent work in the more general setting of string algebras by Eisele, Janssens, and Raedschelders. In my talk, I will discuss a new, simple combinatorial description of $\mathrm{Hom}(X, \tau Y)$ between indecomposable τ-rigid modules in the gentle algebra setting. This allows us to describe the poset of functorially finite torsion classes purely in terms of string combinatorics. We recover some posets which have been studied in the combinatorics literature (in particular, the Tamari lattice, the Grassmann-Tamari posets of Santos-Stump-Welker, and McConville's Grid-Tamari lattices). Representation theory then provides a simple conceptual explanation for the fact that, under good circumstances (which include the aforementioned examples), these posets are lattices. This is joint work with Thomas Brüstle, Guillaume Douville, Kaveh Mousavand, and Emine Yıldırım.

Kent Vashaw (Louisiana State University).
Prime spectra of 2-categories.

We will describe a general theory of prime, completely prime, semiprime, and primitive ideals of 2-categories and $\mathbb{Z}_+$-rings. On the one hand, these notions provide a bridge between prime spectra of noncommutative rings and total positivity. On the other hand, they lead to a natural set of integrality conditions under which a prime ideal of an algebra is categorifiable. As an application of the general theory we obtain categorifications of the coordinate rings of Richardson varieties for arbitrary symmetric Kac-Moody algebras and Dixmier quotients of semisimple Lie algebras (work in progress). This is joint work with Milen Yakimov.

Emily Witt (University of Kansas).
Finding Lyubeznik numbers.

Lyubeznik numbers are integers associated to a local ring that can reveal important geometric properties. We describe some of these properties, as well as some of the methods used to determine them. This includes techniques involving representation and invariant theory, and D-module methods.