Quantum Binary Polyhedral Groups
And Their Actions On Quantum Planes

Chelsea Walton

Joint work with Kenneth Chan, Ellen Kirkman, and James Zhang

November 18, 2012
An investigation of noncommutative/ Hopf invariant theory...
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...quantizations of results in classical invariant theory
Goal

An investigation of noncommutative/ Hopf invariant theory...
...quantizations of results in classical invariant theory

Actions of finite subgroups of $SL_2(\mathbb{C})$

on

“planes” $\mathbb{C}[u, v]$
An investigation of noncommutative/ Hopf invariant theory... 
...quantizations of results in classical invariant theory

Actions of quantum finite subgroups of $SL_2(\mathbb{C})$
on

“quantum planes”: noncommutative $\mathbb{C}[u, v]"
Let’s recall some classical results.

Take $G$ a finite subgroup of $GL_2(k)$ acting faithfully on $k[u, v]$.

Put $k = \mathbb{C}$
Let’s recall some **classical results**.

Take $G$ a finite subgroup of $GL_2(k)$ acting faithfully on $k[u, v]$.

[STC] $k[u, v]^G$ regular?

$k[u, v]^G \cong k[u', v'] \iff G \text{ is generated by reflections.}$
Let’s recall some classical results.

Take $G$ a finite subgroup of $GL_2(k)$ acting faithfully on $k[u, v]$.


$G$ is generated by reflections.

[Watanabe] $k[u, v]^G$ Gorenstein?

$G \leq SL_2(k) \implies k[u, v]^G$ Gorenstein
Let’s recall some classical results.

Take $G$ a finite subgroup of $GL_2(k)$ acting faithfully on $k[u, v]$.

$$[STC] \quad k[u, v]^G \text{ regular?}$$
$$k[u, v]^G \cong k[u', v'] \iff$$
$G$ is generated by reflections.

$$[Watanabe] \quad k[u, v]^G \text{ Gorenstein?}$$
$$G \leq SL_2(k) \implies k[u, v]^G \text{ Gorenstein}$$

$$[Klein] \quad \text{Finite subgroups of } SL_2(k) \text{ are classified up to conjugation.}$$
$$\text{types: } A_n \quad D_n \quad E_6 \quad E_7 \quad E_8$$

“binary polyhedral groups” = $G_{BPG}$

...they are not generated by reflections
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“binary polyhedral groups” $=: G_{BPG}$

...they are not generated by reflections


The “Kleinian” or “DuVal” singularities $X = \text{Spec}(k[u, v]^{G_{BPG}})$ are precisely the rational double points and the resolution graph of $X$ is Dynkin.
Objects of Study

“quantum finite subgroups of $SL_2(k)$” acting on “quantum planes”
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For $q \in k^\times$, categorically–

quantum groups - dual to - Hopf algs

$SL_q(2) \cdots \cdots \cdots \mathcal{O}_q(SL_2(k))$

$G_q$ fin. subgrp \hspace{1cm} $\mathcal{O}_q(G)$ fin. Hopf quot.
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Finite dim’l Hopf algebras $H$

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Finite dim’l Hopf algebras $H$

...that are not necessarily finite quotients of $O_q(SL_2(k))$

with structure: $(H, m, \Delta, u, \varepsilon, S)$
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AS regular algebras $R$ of gldim 2

AS = Artin-Schelter
* $R$ is graded with $R_0 = k$
* global dimension 2
* AS-Gorenstein
* polynomial growth
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Viewed as ‘noncommutative $k[u, v]$’ in Noncommutative Projective AG
Objects of Study

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Classified up to isomorphism:

$k_q[u, v] := k[u, v]/(vu - quv), \ q \in k^\times$

$k_J[u, v] := k[u, v]/(vu - uv - u^2)$
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$H$ acts on $R$ if $R$ is a left $H$-module algebra: $R$ is a left $H$-module and

$h \cdot (ab) = \sum (h_1 \cdot a)(h_2 \cdot b)$ and $h \cdot 1_R = \epsilon(h)1_R$ for all $h \in H$, and for all $a, b \in R$
Let $H \neq k$ be a finite dimensional Hopf algebra acting on an AS regular algebra $R$ of global dimension 2.

(H1) [notion of faithfulness]

(H2) $H$ preserves the grading of $R$

(H3) [notion of $H$-action having ‘determinant 1’]

... as results involving $G$ with $\det(G) = 1$ motivate our results.

See [DuVal-McKay] for instance.
Let $H \neq k$ be a finite dimensional Hopf algebra acting on an AS regular algebra $R$ of global dimension 2.

(H1) $H$ acts on $R$ inner faithfully: there is not an induced action of $H/I$ on $R$ for any nonzero Hopf ideal $I$ of $H$

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(H3) [notion of $H$-action having ‘determinant 1’] ... as results involving $G$ with $\det(G) = 1$ motivate our results.

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(H3) $H$-action of $R$ have trivial “homological determinant”.
here, $\text{hdet}_HR: H \to k$ and it is trivial if equal to the counit map $\epsilon$
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(H3) $H$-action of $R$ have trivial “homological determinant”.

**Definition.** A Hopf algebra $H$ satisfying the conditions above is called a quantum binary polyhedral group, denoted by $H_{QBPG}$. 
Main Result

**Theorem.** [CKWZ] The pairs $(H_{QBPG}, R_{ASreg2})$ are classified as follows.
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**Theorem.** [CKWZ] The pairs $(H_{QBPG}, R_{Asreg2})$ are classified as follows.

- **$H$ noncom & s.s.**
  
  $(kG_{BPG}, k[u, v])$

  $G_{BPG}$ nonabelian

- $(kD_{2n}, k_{-1}[u, v])$
  
  $n \geq 3$

- $(\mathcal{D}(G_{BPG})^\circ, k_{-1}[u, v])$

  $\mathcal{D}(G_{BPG})$: Hopf deformation

  of nonabelian b.p.g. [BN]
Main Result

Theorem. [CKWZ] The pairs \((H_{QBPG}, R_{Asreg2})\) are classified as follows.

<table>
<thead>
<tr>
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<th>Description</th>
</tr>
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<tbody>
<tr>
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<td><em>(kG_{BP}, k[u, v]</em>)&lt;br&gt;(G_{BP}) nonabelian</td>
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<td><em>(kC_n, k_q[u, v])</em>&lt;br&gt;non-diagonal action</td>
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Main Result

**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{AS_{reg}2})\) are classified as follows.

\[
R = k[u, v] \implies H = kG_{BPG}, \text{ no "new" } H
\]

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\[O_q(SL_2)\]
**Main Result**

**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{ASreg})\) are classified as follows.

For \(R = k_{-1}[u, v]\)

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**Main Result**

**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{ASreg2})\) are classified as follows.

For \(R = k_q[u, v]\) with \(q\) a root of unity, \(q^2 \neq 1\)

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**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{ASreg})\) are classified as follows.

For \(R = k_q[u, v]\) for \(q\) not a root of 1

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**Main Result**

**Theorem.** [CKWZ] The pairs \((H_{QBPG}, R_{Asreg2})\) are classified as follows.

For \(R = k_J[u, v]\)

\[
\begin{array}{|c|c|c|}
\hline
H \text{ noncom \& s.s.} & H \text{ comm (\& s.s.)} & H \text{ nonsemisimple} \\
\hline
(kG_{BP}G, k[u, v]) & (kC_2, k_J[u, v]) & \text{For } q \text{ is a root of } 1, q^2 \neq 1 \\
G_{BP}G \text{ nonabelian} & \text{diagonal action} & (T_q, \alpha, n)^\circ, k_{q^{-1}}[u, v]) \\
(kD_{2n}, k_{-1}[u, v]) & (kC_2, k_{-1}[u, v]) & T_{q, \alpha, n}: \text{generalized Taft alg.} \\
n \geq 3 & \text{non-diagonal action} & (H, k_{q^{-1}}[u, v]) \text{ ord}(q) \text{ odd} \\
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\hline
\end{array}
\]
Further Results

Given a pair \((H = H_{QBPG}, R = R_{ASreg2})\) in the main theorem, to say:

a finite dimensional Hopf algebra \(H\) acts inner faithfully and preserves the grading of an AS regular algebra \(R\) of gldim 2, with \(H\)-action having trivial homological determinant

we have the following results.

\[
R^H = \{ r \in R \mid h \cdot r = \epsilon(h)r \text{ for all } h \in H \}
\]

[On the regularity of the invariant subring \(R^H\), motivated by [STC]]

[On the Gorenstein condition for the invariant subring \(R^H\), motivated by [Watanabe]]
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\[R^H = \{ r \in R \mid h \cdot r = \epsilon(h)r \text{ for all } h \in H\}\]

**Theorem.** [CKWZ] Let \((H, R)\) be as above. If \(R^H \neq R\), then \(R^H\) is *not* AS-regular. (\(R^H\) has \(\infty\) gldim.)

[On the Gorenstein condition for the invariant subring \(R^H\), motivated by [Watanabe]]
Given a pair \((H = H_{QBPG}, R = R_{ASreg2})\) in the main theorem, to say:

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we have the following results.

\[ \mathcal{R} = \{ r \in R \mid h \cdot r = \varepsilon(h)r \text{ for all } h \in H \} \]

**Theorem.** [CKWZ] Let \((H, R)\) be as above. If \(\mathcal{R} \neq R\), then \(\mathcal{R}\) is *not* AS-regular. (\(\mathcal{R}\) has \(\infty\) \(\text{gldim}\)).

**Proposition.** [CKWZ] Let \((H, R)\) be as above. The invariant subring \(\mathcal{R}\) is AS-Gorenstein. (semisimple case by [KKZ])
Future Work

(1) Since $R^H$ is Gorenstein and is not regular ...
Motivated by [DuVal-McKay] and others:

Study the geometry of ‘noncommutative Gorenstein singularities’ $R^H$
for $(H, R)$ in the main theorem, particularly with $H$ semisimple.

(2) Motivated by [STC] and others:
Study finite dimensional Hopf algebra actions on AS regular algebras
of gldim 2 with arbitrary homological determinant.

(3) Since AS regular algebras of gldim 3 have been classified...
Study finite dim'l Hopf algebra actions on AS reg. algs of gldim 3.
... AS regular algebras of gldim $> 3$ have not been classified.
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Study finite dim’l Hopf algebra actions on AS reg. algs of gldim 3.

... AS regular algebras of gldim $> 3$ have not been classified
References:


[DuVal-McKay] P. du Val, On isolated singularities of surfaces which do not affect the conditions of adjunction, 1934; J. McKay, Graphs, singularities, and finite groups, 1980.

[Klein] F. Klein, Ueber binäre Formen mit linearen Transformationen in sich selbst., 1875; Vorlesungen über das Ikosaeder und die Auflösung der Gleichungen vom fünften Grade, 1884.

[STC] = [Ben93, Theorem 7.2.1]

[Watanabe] = [Ben93, Theorem 4.6.2]

Thank you for listening!