Errata for Linear Model Theory: With Examples and Exercises by Dale L. Zimmerman

- Page 31, last line before Theorem 2.15.12: Insert "definite" between "nonnegative" and "matrices"
- Page 35, matrix displayed in Definition 2.18.2: Change the subscript on the bottom right matrix from n to k
- Page 51, Theorem 3.3.9(a): Change the dimensions of **U** from $n \times n$ to $p \times p$
- Page 88, Exercise 5.7: In all 5 parts, replace y_{ij} with y_{ijk}
- Page 99, displayed expression for a basis of estimable functions: Remove the duplicate comma after $\mu + \alpha_4 + \gamma_2$
- Page 137, two lines above expression (7.3): Replace $(p p^*)$ with p
- Page 164, fifth line: Replace $i 1, \ldots, n$ with $i = 1, \ldots, n$
- Page 218, Figure 10.1: Replace $\hat{\boldsymbol{\beta}}$ with $\boldsymbol{\check{\beta}}$
- Page 312, Example 13.2-2: In three places in this example, reference is made to Example 11.1-2, but the reference actually should be to Example 11.1-3. Also, the definitions of \mathbf{k}^T and h should be $\mathbf{k}^T = [1/(1-\rho^2)](\rho^{n+t-1}, \rho^{n+t-2}, \dots, \rho^t)$ and $h = 1/(1-\rho^2)$. Finally, after the equals sign in the expression for the prediction error variance, the first term should be $\left(\frac{\sigma^2}{1-\rho^2}\right)$.
- Page 322, third line: There is a duplicate " $\mathbf{R} =$ "
- Page 330, 5th line: Replace "with" with "within"
- Page 337, Exercise 13.19: The third part of this problem, labeled "a.", should be labeled as "(c)" and the fourth part should be labeled as "(d)"
- Page 343, 5th line above Theorem 14.1.4: Remove the extra right parenthesis
- Page 378, 3rd line: Add "with k groups and ν degrees of freedom" after "studentized range distribution"
- Page 393, Definition 15.2.1: Replace the first and third instances of \mathbf{C} with \mathbf{C}^T , and replace the first instance of \mathbf{c}_0 with \mathbf{c}_0^T . Also, replace "s-vector" with "s-dimensional row vector"
- Page 393, 8th line after the end of Definition 15.2.1: Replace C with C^T
- Page 400, heading of Example 15.2-2: Replace "Nonzero" with "Unequal"
- Page 404, heading of Example 15.2-3: Replace the second instance of "Nonzero" with "Unequal"
- Page 407, first sentence: This sentence is out of place; it should appear lower on the page, immediately prior to the sentence that begins with "By this last result, ..."
- Page 410, 2nd and 3rd lines: Replace "bivariate random vectors $(A_1, B_1)^T$ and $(A_2, B_2)^T$ " with "trivariate random vectors $(A_1, \tau_1, B_1)^T$ and $(A_2, \tau_2, B_2)^T$ "

- Page 411, line 5: Insert "the" immediately before "Bonferroni"
- Page 415, 6th line: Replace "intervals" with "functions"
- page 415, 8th line of Point 4: Insert "may" immediately before "be"
- Page 416, 14th line: After the period, add "We consider only cases for which $\operatorname{var}[(\hat{\psi}_1, \hat{\psi}_2, \dots, \hat{\psi}_k)^T] = c^2 \sigma^2 \mathbf{I}$ for some positive real number c."
- Page 453, last line: Strictly, θ_m cannot be set equal to 0 because such a value does not belong to the parameter space. So instead of choosing θ to be each of the unit *m*-vectors, one may first choose θ to be $\mathbf{u}_m^{(m)}$ to establish that $f_m = \operatorname{tr}(\mathbf{A})$, and then choose θ to be formed by appending 1 to the bottom of each of the unit (m-1)-vectors, to establish that $\operatorname{tr}(\mathbf{Z}_i^T \mathbf{A} \mathbf{Z}_i) = f_i \ (i = 1, \dots, m-1).$
- Page 466, 7th line: Replace the expression for U_2 with $n_2 S_2^2/\alpha_2^2$
- Page 487, displayed expression in 4th line: Replace $\mathbf{v}_{uu}(\boldsymbol{\theta})$ with $v_{uu}(\boldsymbol{\theta})$
- Page 489, first displayed expression in Example 17.1-1: Replace \bar{y} with \bar{y} ...
- Page 490, first line of last paragraph of Example 17.1-1: Replace S_1^2 and S_2^2 with $\hat{\sigma}^2$ and $\hat{\sigma}_b^2$, respectively

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- Page 11, solution to Exercise 3.5: In the next to last line, replace $k \neq 1$ with $k \neq -1$.
- Page 33, Exercise 5.7: In all 5 parts, replace y_{ij} with y_{ijk} .
- Page 49, solution to Exercise 6.16: Replace parts (a), (b), (c), and (d) with the following:
 (a) Any ψ_{iji'j'} ≡ ξ_{ij} ξ_{ij'} ξ_{i'j} + ξ_{i'j'} = μ_{ij} μ_{ij'} μ_{i'j} + μ_{i'j'} is estimable if the four cell means included in its definition are estimable. Thus, if there are no empty cells, all ψ_{iji'j'} for i ≠ i' and j ≠ j' are estimable. A basis for this set of functions is {ψ₁₁₂₂, ψ₁₂₂₃, ψ₂₁₃₂, ψ₂₂₃₃} because they are clearly linearly independent and

$\psi_{1122} + \psi_{1223}$	=	$\xi_{11} - \xi_{12} - \xi_{21} + \xi_{22} + \xi_{12} - \xi_{13} - \xi_{22} + \xi_{23} = \psi_{1123},$
$\psi_{1122} + \psi_{2132}$	=	$\xi_{11} - \xi_{12} - \xi_{21} + \xi_{22} + \xi_{21} - \xi_{22} - \xi_{31} + \xi_{32} = \psi_{1132},$
$\psi_{1223} + \psi_{2233}$	=	$\xi_{12} - \xi_{13} - \xi_{22} + \xi_{23} + \xi_{22} - \xi_{23} - \xi_{32} + \xi_{33} = \psi_{1233},$
$\psi_{2132} + \psi_{2233}$	=	$\xi_{21} - \xi_{22} - \xi_{31} + \xi_{32} + \xi_{22} - \xi_{23} - \xi_{32} + \xi_{33} = \psi_{2133},$

and

$$\psi_{1122} - \psi_{1223} - \psi_{2132} + \psi_{2233} = \xi_{11} - \xi_{12} - \xi_{21} + \xi_{22} + \xi_{12} - \xi_{13} - \xi_{22} + \xi_{23} + \xi_{21} - \xi_{22} - \xi_{31} + \xi_{32} + \xi_{22} - \xi_{23} - \xi_{32} + \xi_{33} = \psi_{1133}.$$

(b) The location of the empty cell determines which interaction contrasts are estimable but does not determine the number of linearly independent interaction contrasts because we can always perform row and column permutations to put the empty cell in any row and column we desire. So without loss of generality assume that the empty cell is (3,3). Then a basis for the estimable interaction contrasts is $\{\psi_{1122}, \psi_{1223}, \psi_{2132}\}$. The basis will change depending on which cell is empty, but its dimension is 3 regardless of which cell is empty. There are five additional linearly independent estimable functions, which are not interaction contrasts, as follows: $E(y_{11}) = \mu + \alpha_1 + \gamma_1 + \xi_{11}$, $E(y_{21} - y_{11}) = \alpha_2 - \alpha_1 + \xi_{21} - \xi_{11}$, $E(y_{31} - y_{11}) = \alpha_3 - \alpha_1 + \xi_{31} - \xi_{11}$, $E(y_{12} - y_{11}) = \gamma_2 - \gamma_1 + \xi_{12} - \xi_{11}$, $E(y_{13} - y_{11}) = \gamma_3 - \gamma_1 + \xi_{13} - \xi_{11}$. Thus, rank(\mathbf{X}) = 8, regardless of which cell is empty.

(c) Either the two empty cells are in the same row or column, or they are in different rows and columns. The "same row" case is equivalent to the "same column" case. For this case, again by considering row and column permutations, without loss of generality we may assume cells (2,3) and (3,3) are empty. Then a basis for the estimable interaction contrasts is $\{\xi_{1122}, \xi_{2132}\}$. The basis will change depending on which two cells in the same row or column are empty, but its dimension is 2 regardless. There are five additional linearly independent estimable functions, which are not interaction contrasts, as follows: $E(y_{11}) = \mu + \alpha_1 + \gamma_1 + \xi_{11}$, $E(y_{21} - y_{11}) = \alpha_2 - \alpha_1 + \xi_{21} - \xi_{11}$, $E(y_{31} - y_{11}) = \alpha_3 - \alpha_1 + \xi_{31} - \xi_{11}$, $E(y_{12} - y_{11}) = \gamma_2 - \gamma_1 + \xi_{12} - \xi_{11}$, $E(y_{13} - y_{11}) = \gamma_3 - \gamma_1 + \xi_{13} - \xi_{11}$. Thus, rank(\mathbf{X}) = 7, regardless of which cell is empty.

On the other hand, if the empty cells lie in different rows and columns, which without loss of generality we may take to be cells (1,1) and (3,3), then a basis for the estimable interaction contrasts is $\{\psi_{1223}, \psi_{2132}\}$. Again, the basis will change depending on which two cells are empty, but its dimension is 2 regardless. There are again five additional linearly independent estimable functions, which are not interaction contrasts, as follows: $E(y_{12}) = \mu + \alpha_1 + \gamma_2 + \xi_{12}$, $E(y_{22} - y_{12}) = \alpha_2 - \alpha_1 + \xi_{22} - \xi_{12}$, $E(y_{32} - y_{12}) = \alpha_3 - \alpha_1 + \xi_{32} - \xi_{12}$, $E(y_{22} - y_{21}) = \gamma_2 - \gamma_1 + \xi_{22} - \xi_{21}$, $E(y_{23} - y_{21}) = \gamma_3 - \gamma_1 + \xi_{23} - \xi_{21}$. Thus, rank(\mathbf{X}) = 7 as in the previous case, regardless of which cell is empty.

(d) No entire row and no entire column in the 3×3 layout can be empty. The maximum allowable number of empty cells for this to be true and for there to be at least one estimable interaction contrast is 4. Without loss of generality assume that cells (1,3), (2,3), (3,1), and (3,2) are empty. Then the only estimable interaction contrast is ψ_{1122} . The lone estimable contrast will change, of course, depending on which four cells are empty, but there is always one such contrast. There are four additional linearly independent estimable functions, which are not interaction contrasts, as follows: $E(y_{11}) = \mu + \alpha_1 + \gamma_1 + \xi_{11}$, $E(y_{21} - y_{11}) = \alpha_2 - \alpha_1 + \xi_{21} - \xi_{11}$, $E(y_{12} - y_{11}) = \gamma_2 - \gamma_1 + \xi_{12} - \xi_{11}$, $E(y_{33}) = \mu + \alpha_3 + \gamma_3 + \xi_{33}$. Thus, rank(\mathbf{X}) = 5, regardless of which cell is empty.

- Page 67, solution to Exercise 7.8: The expression for the least squares estimator of β_2 should be SXY/SXX.
- Page 98, solution to Exercise 8.5d: The entry in the Residual row and Rank column of the ANOVA table should be q(q-2).
- Pages 126 and 127, ANOVA tables: The entry in the Residual row and Rank column should be qm q m, not qm q m 1.
- Page 260, first line of part (a) of solution to Exercise 15.5: Replace "Because $\hat{\boldsymbol{\beta}}_{-1}$ is unbiased for $\boldsymbol{\beta} \dots$ " with "Because $\mathbf{x}_1^T \hat{\boldsymbol{\beta}}_{-1}$ is unbiased for $\mathbf{x}^T \boldsymbol{\beta} \dots$ "

- Page 260, next-to-last line: Replace $\hat{\mathbf{e}}_{1,-1}$ with $\hat{e}_{1,-1}$. Make this same change in the third and fourth lines of page 261.
- Page 261, line 4: Delete the equals sign and the mathematical expression that precedes it.
- Page 261, line 9: Replace n p 1 with $n p^* 1$.
- Page 293, line 2: Replace the given **C** with

NOTE: These lists are current as of 02/13/2025.