To solve $x^2 - 5 = 0$ for the root $\alpha = \sqrt{5}$, begin with

$$x = x + c\left(x^2 - 5\right) \equiv g(x)$$

Then

$$g'(x) = 1 + 2cx$$
$$g'\left(\sqrt{5}\right) = 1 + 2c\sqrt{5}$$

Choose c so that $g'\left(\sqrt{5}\right) \approx 0$,

$$c \approx \frac{-1}{2\sqrt{5}} \doteq -0.22361$$

Using $c = -\frac{1}{4}$, we have the iteration

$$x_{n+1} = x_n - \frac{1}{4}\left(x_n^2 - 5\right), \qquad n \geq 0$$

The rate of linear convergence is given by

$$g'\left(\sqrt{5}\right) = 1 - \frac{1}{2}\sqrt{5} \doteq -0.11803$$

n	x_n	$\alpha - x_n$	*Ratio*
0	2.0	$2.361E-1$	
1	2.25	$-1.393E-2$	$-.0590$
2	2.234375	$1.693E-3$	$-.1215$
3	2.2362709	$-1.991E-4$	$-.1176$
4	2.236044466	$2.351E-5$	$-.1181$
5	2.236070753	$-2.276E-6$	-1181