Building Verified Language Tools in Operational Type Theory

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From Meta-Theory to Tools

- Mechanized meta-theory great.
- Verified language tools also great!
- The combination definitely the greatest.
Meta-theory and Tools for LF

- Paper meta-theory for LF [Harper+05],[Watkins+02].
- Machine-checked meta-theory for LF [Urban+08].
- Unverified tools for LF: TWELF, FLIT, SC, LFSC.
- Verified tool (this talk): GOLFSOCK.
  - Verify that optimized LF checker builds type-correct LF.
  - Uses a declarative presentation of LF.
  - Still efficient, but much more trustworthy.
  - Partial verification.
Basic idea: interleave parsing and checking [Stump08]. Combine with bidirectional type checking.

- Synthesizing: $\Gamma \vdash t \Rightarrow T$.
- Checking: $\Gamma \vdash t \Leftarrow T$.

ASTs built for subterms iff they will appear in the type $T$. E.g.,

$$(\text{refl } x+y) \Rightarrow x+y \equiv x+y$$

- AST must be built for $x+y$.
- But not $(\text{refl } x+y)$.

C++ implementation: small footprint, fastest checker I know.
A Need for Correctness

- LF with Side Conditions (LFSC) proposed for SMT.
  - Satisfiability Modulo Theories.
  - SMT solvers check large formulas, produce big proofs.
  - Must check proofs efficiently.
  - LFSC provides flexible intermediate proof language.
  - Extends LF with computational side conditions.

- Problems with C++ checker:
  - Lack of memory safety => many days with valgrind.
  - Optimizations reduce trustworthiness.

- As features are added to checker, trust diminishes.

- Additional assurance is required.
Towards A Verified LFSC Checker

**GOLFSOCK (“GURU LFSC”).**
- **GURU** is a verified functional programming language.
- Supports mutable state, non-termination, I/O.
- Verification via dependent types, induction proofs.
- Type/proof checker, compiler to efficient C code.
- Beating native code OCAML on small testcases.

**Status:**
- Incremental LF checking implemented.
- Running reasonably fast: 50% slower than C++ version.
- Specification: ASTs we build are type correct LF.
- Expressed with dependent types, declarative LF.
- 4300 lines code, proof; 13000 lines standard library (e.g., tries).
GURU and Operational Type Theory

GURU implements Operational Type Theory (OpTT).

OpTT is new type theory intended to:

▶ Combine programming, theorem proving (cf. ATS, Epigram, Ynot).
▶ Allow general recursion, other effects.
▶ Retain sound logic.
▶ Retain decidability of type checking.
▶ Support external reasoning about dependently typed programs.
▶ Support compilation to efficient executables.

Critical design idea: separate different reductions.

▶ Reduction for definitional equality ($\equiv$).
▶ Reduction for programs.
▶ Normalization (aka, cut elimination) for proofs.
Rejection of Curry-Howard

- Proofs ≠ Programs, Formulas ≠ Types.

```
```

```
forall (x : nat). truei    :   Forall (x : nat). True
```

- Otherwise non-terminating programs = unsound proofs.
Rejection of Conversion

- Definitional equality ($\equiv$) cannot include program reduction.
- Otherwise type checking undecidable.
- Adopt a very weak $\equiv$ ($\equiv_\alpha$, definitions, sugar).
- Contrast with strong conversion relations.
  - CIC: $\equiv$ includes $\equiv_\beta$, terminating recursion.
  - CCIC: $\equiv$ uses decision procedures, hypotheses.
- With conversion, lose definitional transparency.
- Typing holds modulo $\equiv$, but not other operations.
  - $\Gamma \vdash t : T$ $\Rightarrow$ $\Gamma' \vdash t' : T'$ with $\Gamma \equiv \Gamma'$, $t \equiv t'$, $T \equiv T'$.
  - Rewriting modulo $\equiv_\beta$ only recently decidable [Stirling06].
  - In Coq, many tactics do not work modulo $\equiv$.
  - In GURU, all tactics work modulo $\equiv$. 
Operational Equality

- Due to weak $\equiv$, need casts in code (and proofs):

$$
\begin{align*}
\text{cast } t \text{ by } P : T_2 \\
\end{align*}
$$

- Reasoning about code with casts tedious in other systems.

- In OpTT, reason about unannotated programs.

  - Propositional equality $\{ t = t' \}$ holds if $t \downarrow t'$.
  - No type annotations, casts, proofs in $t, t'$.
  - No *specification*al data.
  - Vastly simplifies external reasoning about code.
  - Annotations dropped by definitional equality.
Example: Vector Append

Inductive vec : Fun(A:type)(n:nat).type :=
   vecn : Fun(A:type).<vec A Z>
   <vec A (S n)>.

vec_append : Fun(A:type)(spec n m:nat)
   <vec A (plus n m)>

vec_append_assoc :
  Forall(A:type)(n1 : nat)(l1 : <vec A n1>)
  { (vec_append (vec_append l1 l2) l3) =
    (vec_append l1 (vec_append l2 l3)) }
Functional Modeling and Ownership

- Following [Swierstra+07]: awkwardness => modeling school.
- Awkward squad modeled functionally.
  - Standard input is a list of chars.
  - `getc()` is head.
  - Mutable arrays of length $n$ are vectors of length $n$.
  - `read` and `write` are pure, $O(n)$ operations.
- Reason about code using functional model.
- Replace during compilation with non-functional implementation.
- Restrict usage for soundness (monads or linear types).
- **GURU** uses linear types.
  - Fit well with ownership types.
  - **GURU** statically tracks ownership of all data.
  - Enables reference counting for memory management.
  - Function inputs `unowned`, `owned`, `unique`, or `unique_owned`. 
**GOLFSOCK: Symbols**

- Incrementally consume textual input, type check LF.
- LF variables (constants) implemented as 32-bit words.
  - Implementation with `nat` too slow.
  - Words are functionally modeled as `<vec bool 32>`.
  - Trusted operations: increment, equality check, create 0.
  - Reason via model, also via conversion to `nat`.
- Symbol table maps strings (lists of chars) to (var, type) pairs.
- Symbol table implemented as a trie.
- Mutable char-indexed arrays of subtries at each node.
GOLFSOCK: LF derivations

- Code “builds” specificational LF derivations.
- For $\Gamma \vdash t \leftrightarrow T$ (or $\Gamma \vdash t \Rightarrow T$), we build $\langle \text{deriv } G \ t \ T \rangle$.
- Context encoded as a list of (var,type) pairs.
- Must map the symbol table to context.
  - Difficult.
  - Must prove lemmas like trie membership $\Rightarrow$ context membership.
  - Resulting context is not ordered.
  - Phrase typing rules for unordered contexts.
  - Ok, because vars uniquely named.
Empirical Results

<table>
<thead>
<tr>
<th>benchmark</th>
<th>size (MB)</th>
<th>C++ impl</th>
<th>GOLFSOCK</th>
<th>TWELF</th>
</tr>
</thead>
<tbody>
<tr>
<td>cnt01e</td>
<td>2.6</td>
<td>1.3</td>
<td>2.0</td>
<td>14.0</td>
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<td>tree-exa2-10</td>
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<td>1.7</td>
<td>2.5</td>
<td>18.6</td>
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<td>5.8</td>
<td>8.8</td>
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<td>10.0</td>
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<td>timeout</td>
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<td>89.7</td>
<td>exception</td>
</tr>
</tbody>
</table>

Figure: Checking times in seconds for QBF benchmarks

- Good, since some optimizations not implemented.
Conclusion

- **GOLFSOCK**: towards verified, efficient language tools.
- OpTT makes this easier:
  - Not required *a priori* to prove termination.
  - Reason about code with annotations dropped.
  - Use dependent types for big functions *(check, 1200 lines).*
  - Supports functional modeling.
- Onward towards verified, efficient software!

[www.guru-lang.org](http://www.guru-lang.org)