TRELLYS and Beyond: Type Systems for Advanced Functional Programming

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What is Functional Programming?

Central Ideas:
- Functions as data (higher-order functions, partial applications).
  - `List.map ((+) 10) [1 ; 2 ; 3]`
- Procedures behave like mathematical functions.
  - monads for making effectful procedures behave.

Culture:
- memory safety (via types)
- compile-time debugging via types
- type inference
- garbage collection
- pattern matching, inductive data
- module systems
The State of FP

Practical  ↔  FP  ↔  Verifiable

- Software Transactional Memory
- Lightweight threads
- Continuing compiler improvements
- GADTs, dependent types
- Resource typing (HTT)
- More powerful type inference

Sweet spot, or missing all targets?
This Talk

- **Verifiability: TRELLYS**
  - practical dependently typed language.
  - quasi-implicit products.
  - briefly: effect system for termination, termination casts.

- **Practicality: BLAISE**
  - goal: memory-safe functional programming without GC.
  - idea: divide pointers into primary and alias.
  - primary pointers used linearly.
  - alias pointers must have reciprocal backpointer.
  - *early-stage work.*
Design Goals and Design Ideas for TRELLYS
The TRELLYS Project

Goal: a new functional language with dependent types.

- Why Dependent Types Matter™:
  - incremental verification.
  - standard example:

\[
\begin{align*}
[10; 20; 30] &: \text{vec int 3} \\
\text{append} &: \text{Forall}(A:\text{type})(n1 \ n2 : \text{nat})
               \quad l1 : \text{vec} \ A \ n1 \to l2 : \text{vec} \ A \ n2 \to \text{vec} \ A \ n1+n2
\end{align*}
\]

  - small intellectual step from list A to vec A n.
  - other formal methods: much bigger leap!

- TRELLYS: combine resources to build compiler, libraries, etc.
- Involve broader community (working groups).
- First step: devise core language (PIs + SPJ, CM, BB, WS).
Goals for TRELLYS Core Language

- CBV, mutable state, inductive types, gen. recursion.
- Straightforward syntax-directed type checking.
  - surface language: type inference, automated reasoning.
  - elaboration to core adds annotations.
  - no fancy algorithms in the core.
  - trustworthy type checking for core.
- Logically sound fragment under Curry-Howard.
  - general recursion => unsound proof.
  - want a sound notion of proof.
  - without proofs, would need runtime checks.
Technical Goals for TRELLYS Core Language

- General recursive functions, sound proofs.
- Implicit products.
  - specificational ("ghost") data.
    
    ```
    l1 : vec A n1 -> l2 : vec A n2 -> vec A n1+n2
    ```
  - not computationally relevant.
  - dropped during compilation, and formal reasoning.

- `type : type`
  - most powerful form of polymorphism known.
  - allows type-level computation, data structures:
    ```
    [ int ; bool ; int -> int ] : list type
    ```

- Non-constructive reasoning (AS).
  - already distinguishing terminating/general recursive.
  - so distinguish proofs and programs.
  - motivation: case-split on termination of term.
  - cf. Logic-Enriched Type Theory, Zhaohui Luo.
Aside: Non-Constructive Reasoning

- Suppose quantifiers range over values.
- Define `foldr`:
  
  \[
  (\text{foldr } f \ b \ []) = b \\
  (\text{foldr } f \ b \ (x:xs)) = (f \ x \ (\text{foldr } f \ b \ xs))
  \]

- Suppose \( u \) is an assumption of:
  
  \[
  \text{Forall}(a:A)(b:B). \ (f \ a \ b) = (g \ a \ b)
  \]

- Then:
  
  \[
  (\text{foldr } f \ b \ l) = (\text{foldr } g \ b \ l)
  \]

- Proof by induction on \( l \). Step case:
  
  \[
  (\text{foldr } f \ b \ (a:l')) = (f \ a \ (\text{foldr } f \ b \ l')) \quad \text{// evaluation} \\
  = (f \ a \ (\text{foldr } g \ b \ l')) \quad \text{// IH} \\
  = (g \ a \ (\text{foldr } g \ b \ l')) \quad \text{// not allowed by \( u \)!} \\
  = (\text{foldr } g \ b \ (a:l'))
  \]

- Solution: case split on whether or not \( (\text{foldr } g \ b \ l') \) terminates.
Implicit Products

- Want **erasure**: \[ \text{append } A \ n_1 \ n_2 \ l_1 \ l_2 \] = \[ \text{append } l_1 \ l_2 \].
- For compilation, and for formal reasoning.
- General theoretical approach:
  1. define unannotated system:
     - terms \( t \), types \( T \).
     - type assignment \( \Gamma \vdash t : T \), reduction \( t \rightsquigarrow t' \).
     - type assignment may not be algorithmic.
  2. do metatheory for unannotated system.
  3. define annotated system:
     - annotated terms \( a \), types \( A \).
     - algorithmic typing \( \Gamma \triangleright a : A \).
     - erasure \( |a| = t, |A| = T \).
  4. conclude metatheory for annotated system.
Example: $T^{vec}$

- Gödel’s System T + equality types, vector types.
- Unannotated terms:
  
  $t ::= x | (t \, t') | \lambda x.\, t | 0 | (S \, t) | (R_{\text{nat}} \, t \, t' \, t'') | \text{nil} | (\text{cons} \, t \, t') | (R_{\text{vec}} \, t \, t' \, t'') | \text{join}$

- Reduction relation $t \leadsto t'$ as expected.
- Unannotated types:
  
  $T ::= \text{nat} | \langle \text{vec} \, T \, t \rangle | \Pi x : T.\, T' | \forall x : T.\, T' | t = t'$

- $\forall x : T.\, T'$ – implicit product.
Type Assignment: Implicit Products

- A form of intersection type.
- Studied for Implicit Calculus of Constructions [Miquel01].
- No term constructs in unannotated system.

\[ \Gamma, x : T' \vdash t : T \quad x \notin FV(t) \quad \frac{\Gamma \vdash t : \forall x : T'.T}{\Gamma \vdash t : \forall x : T'.T} \quad \frac{\Gamma \vdash t' : T'}{\Gamma \vdash t : [t'/x]T} \]
Type Assignment: Equality Proofs

- Do not rely on an algorithmic conversion relation.
- Use explicit casts to change types of terms.
- Casts are computationally irrelevant.
- So no term construct for unannotated system.

\[
\begin{align*}
t & \downarrow t' & \Gamma \text{ Ok} & \quad \Gamma \vdash t = t' \\
\Gamma \vdash \text{join}: t = t' \\
\Gamma \vdash t : t_1 = t_2 & \quad \Gamma \vdash t' : [t_1/x]T & x \not\in \text{dom}(\Gamma) & \quad \Gamma \vdash t' : [t_2/x]T
\end{align*}
\]
Annotated $\mathcal{T}^{\text{vec}}$

$$a ::= \ldots \mid (a \ a')^{-} \mid \lambda x : A.a \mid (\text{join } a \ a') \mid (\text{cast } x.A \ a \ a')$$

$$A ::= \text{nat} \mid \langle \text{vec } A \ a \rangle \mid \Pi x : A.A' \mid \forall x : A.A' \mid a = a'$$

$$\begin{align*}
|(a \ a')^{-}| & = |a| \\
|\lambda x : A.a| & = |a| \\
|(\text{cast } x.A \ a \ a')| & = |a'| \\
|(\text{join } a \ a')| & = \text{join}
\end{align*}$$

$$\frac{\Gamma \vdash a : A \quad \Gamma \vdash a' : A' \quad |a| \downarrow |a'|}{\Gamma \vdash (\text{join } a \ a') : a = a'} \quad \frac{\Gamma \vdash a : a_1 = a_2 \quad \Gamma \vdash a' : [a_1/x]A}{\Gamma \vdash (\text{cast } x.A \ a \ a') : [a_2/x]A}$$
Metatheory

- For unannotated system:
  - Standard results: type preservation, progress.
  - Strong normalization via reducibility argument.
- Lifting results to annotated system straightforward.
Large Eliminations

- Type-level computation.
- Many feel essential to dependent types.

\[
T ::= \ldots \mid R\; t\; T\; (\alpha.T')
\]

\[
\frac{\Gamma \vdash t : R\; 0\; T\; (\alpha.T')}{\Gamma \vdash t : T} \quad \frac{\Gamma \vdash t : R\; (S\; t')\; T\; (\alpha.T') \quad \Gamma \vdash t' : \text{nat}}{\Gamma \vdash t : [R\; t'\; T\; (\alpha.T')/\alpha]T'}
\]
Problem: Inconsistent Contexts

- Well-known issue if can equate types:
  \[ u : \text{nat} = (\Pi x : \text{nat}.\text{nat}) \vdash (0 \ 0) : \text{nat} \]

- Can also type diverging terms.
- Same problem arises with large eliminations.
- Problem is even worse with implicit products:
  \[ u : \text{nat} = (\Pi x : \text{nat}.\text{nat}) \vdash (0 \ 0) : \text{nat} \]
  \[ \vdash (0 \ 0) : \forall u : \text{nat} = (\Pi x : \text{nat}.\text{nat}).\text{nat} \]

- Stuck terms would be typable in empty context!
- Solution: quasi-implicit products.
Quasi-Implicit Products

- Idea: do not completely erase implicit abstraction, application.
- Unannotated system:

\[
\begin{align*}
\text{let } t & \text{ ::= } \ldots \mid (\lambda t) \mid t \\
\end{align*}
\]

\[
\begin{align*}
\Gamma, x : T' \vdash t : T \quad x \not\in \text{FV}(t) & \quad \Gamma \vdash (\lambda t) : \forall x : T' . T \\
\Gamma \vdash t : \forall x : T' . T \quad \Gamma \vdash t' : T' & \quad \Gamma \vdash (t) : [t'/x] T
\end{align*}
\]

- Do not reduce beneath quasi-implicit abstraction.
- Meta-theory (including SN) preserved.
- See “Equality, Quasi-Implicit Products, and Large Eliminations” [pending]
Another Design Sketch: $T^{eq\downarrow}$

- “Termination Casts: A Flexible Approach to Termination with General Recursion” [PAR’10].
- Type-and-effect system for termination/possible divergence.
- Include equality types, termination types.
- Termination types reflect the termination effect.
- Terms and types:

\[
\begin{align*}
\theta & ::= \downarrow \mid ? \\
A & ::= \text{nat} \mid \Pi^\theta x : A.A' \mid a = a' \mid \text{Terminates } a \\
0 & ::= y \mid a a' \mid \lambda y : A.a \mid S a \\
S & ::= \text{rec}_{\text{nat}} g(y \ p) : A = a \mid \text{rec } g(y : A) : A' = a \mid \text{case } x.A a a' a'' \\
& \mid \text{join } a a' \mid \text{cast } x.A a' a \mid \text{terminates } a \mid \text{reflect } a a' \mid \text{inv } a a' \\
& \mid \text{contra } A a \mid \text{abort } A
\end{align*}
\]
Termination Casts

- Used to change the effect for a term.
- Proofs of termination first-class.
- External vs. internal verification:
  - Internal: type the function as total.
    \[
    \text{plus} : \Pi x : \text{nat} \Pi y : \text{nat.nat} \downarrow
    \]
  - External: write a proof that the function is total.
    \[
    \text{plus} : \Pi x ? : \text{nat} \Pi y ? : \text{nat.nat} ?
    \]
    \[
    \text{plus\_tot} : \Pi x \downarrow : \text{nat} \Pi y \downarrow : \text{nat.Terminates} (\text{plus} x y) \downarrow
    \]
- $\mathbb{T}_{\text{eq}} \downarrow$ supports both.
Selected Typing Rules ($\Gamma \vdash a : A \theta$)

\[
\begin{align*}
\Gamma \vdash a : A \theta & \quad \Gamma \vdash a' : \text{Terminates } a \downarrow \\
\Gamma \vdash \text{reflect } a a' : A \theta' & \quad \text{AT\_REFLECT} \\
\Gamma \vdash a : A \downarrow & \\
\Gamma \vdash \text{terminates } a : \text{Terminates } a \theta & \quad \text{AT\_REIFY} \\
p \not\in \text{fv} | a | \\
\Gamma' = \Gamma, g : \Pi ? x : \text{nat}. A, y : \text{nat} \\
\Gamma'' = \Gamma', p : \Pi x_1 : \text{nat}. \Pi u : y = S x_1. \text{Terminates } (f x_1) \\
\Gamma'' \vdash a : [y / x] A \downarrow \\
\Gamma \vdash \text{rec}_{\text{nat}} g(y p) : A = a : \Pi x : \text{nat}. A \theta
\end{align*}
\]

AT\_REC\_NAT
Taming Aliased Structures with Bidirectional Pointers: BLAISE
Eliminating Garbage Collection

- GC pros: greater productivity, fewer bugs.
- GC cons: performance hit, unpredictability, complexity.
- Hard to use for real-time systems.
- Resource typing to the rescue?
  - statically track resources.
  - ensure no double deletes, no leaks.
  - alias types, \( L^3 \), HTT [Cornell/Harvard PL].
  - Stateful views [Xi et al.]

Explicitly modeling locations is pretty heavy.

Alternative: find and enforce safe abstractions for aliased structures.
A Safe Abstraction: Bidirectional Pointers

- Idea: divide reference graph into primary/alias pointers.
  - primary pointers should form spanning trees.
  - alias pointers for all other edges.
- Each alias pointer must have a reciprocal backpointer.

To delete a cell:
  - only delete via a primary pointer.
  - must first disconnect all alias pointers.
  - for disconnected alias pointer: must patch up reciprocal pointer.

- Overhead reasonable: one extra word per alias pointer.
- To enforce this abstraction: symbolic simulation.
Example: FIFO Queues

type node a

type queue a

inode : all a . Cell (d : a , n : node a) . node a

enode : all a . Cell (d : a , h : queue a) . node a

mk_queue : all a . Cell (h : node a , t : node a) . queue a

empty_queue : all a . Cell () . queue a
Symbolic Simulation by Example

fun insert<a>(consume d : a ,
    update q : queue a).

(case q of
    empty_queue ->
        let e = new enode <a> in
        e.d = d;
        update q to mk_queue <a>;
        connect e.h q.t;
        q.h = e
    | mk_queue ->
        * let x = disconnect q.t in
        let e = new enode <a> in
        e.d = d;
        connect e.h q.t;
        let y = (take x.d) in
        update x to inode <a>;
        x.d = y;
        x.n = e); 0
fun insert<a>(consume d : a, 
    update q : queue a).

(case q of
    empty_queue ->
        let e = new enode <a> in
        e.d = d;
        update q to mk_queue <a>;
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        connect e.h q.t;
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        update x to inode <a>;
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            update q to mk_queue <a>  
            connect e.h q.t;  
            q.h = e  
| mk_queue ->  
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        let e = new enode <a> in  
            e.d = d;  
            connect e.h q.t;  
        let y = (take x.d) in  
        update x to inode <a>  
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        q.h = e
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        let e = new enode <a> in
            e.d = d;
        connect e.h q.t;
        * let y = (take x.d) in
        update x to inode <a>;
            x.d = y;
            x.n = e); 0
Symbolic Simulation by Example

fun insert<\text{a}>(\text{consume} \ d : \ \text{a},
   \text{update} \ q : \ \text{queue} \ \text{a}).

(case \ q \ of
  \text{empty\_queue} \ ->
    \text{let} \ e = \text{new enode} <\text{a}> \ \text{in}
    \ e.d = d;
    \text{update} \ q \ \text{to} \ \text{mk\_queue} <\text{a}>;
    \text{connect} \ e.h \ q.t;
    \text{q.h} = \ e
  | \text{mk\_queue} \ ->
    \text{let} \ x = \text{disconnect} \ q.t \ \text{in}
    \text{let} \ e = \text{new enode} <\text{a}> \ \text{in}
    \ e.d = d;
    \text{connect} \ e.h \ q.t;
    \text{let} \ y = (\text{take} \ x.d) \ \text{in}
    \text{update} \ x \ \text{to} \ \text{inode} <\text{a}>;
    x.d = y;
    x.n = e); 0

*
Conclusion

**FP and Verifiability: TRELLYS:**
- combine dependent types, general recursion.
- quasi-implicit products.
- termination casts.
- *next step:* putting it all together, with `type: type`.

**FP and Practicality: BLAISE:**
- programming discipline: primary/alias pointers, reciprocal alias pointers.
- enforce by symbolic simulation.
- *next step:* define symbolic simulation, implement.

For more info:
- “Equality, Quasi-Implicit Products, and Large Eliminations”
- “Termination Casts: A Flexible Approach to Termination with General Recursion”
- See also QA9 (blog).

Thanks for having me!