Fast and Flexible Proof Checking for SMT

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An SMT Proof System

- Solver generates formal proofs of unsatisfiability
  - Solver is not trusted
  - Answers can be trusted by verifying the proofs
- Verifier checks the proofs against the axioms and the formulas
  - Trusted component
  - Challenge 1: flexible to accommodate variety of solvers
  - Challenge 2: fast enough for practical usage

CLSAT

- CLSAT 1.0
  - SAT solver w/ proof generation
  - SMT solver (QF_IDL)
  - New: proof generation for SMT
- Proof Formalism
  - Based on Edinburgh Logical Framework (LF)
  - LF is a simple meta logic
  - SMT syntax and axioms defined in LF

LF and LFSC

LF

- LF is based on type theory
- Looks like a functional programming language
- Type computation and checking

LFSC (LF with Side Conditions)

- LF lacks looping and recursion
  - No complicated pattern matching and term building
- LFSC extends LF for
  - Computational side conditions
  - Built-in integer type and arithmetic

A Theorem of Unsatisfiability

$$\Gamma, f : \Phi \vdash t : \mathit{false}$$

- Γ : SMT syntax and axioms
  - 61 rules (32 for CNF conversion, 17 for IDL)
  - 897 lines in LFSC
- $f : \Phi$: assumption of the input formula
- t : the proof statement from the solver
  - Mostly, ≤ 200 MB for benchmarks solved ≤ 900s
  - But, a few of them are greater than 1GB
  - Overhead of proof production was less than 10%

Proof Encoding in LF

$$\dfrac{\begin{matrix}\Gamma \\ \vdots P_1 \\ \Phi_1 \wedge \Phi_2\end{matrix} \qquad \begin{matrix}\Gamma, f_1 : \Phi_1, f_2 : \Phi_2 \\ \vdots P_2 \\ \mathit{false}\end{matrix}}{\mathit{false}} \wedge_e$$

$$\Rightarrow (\mathit{and\_e}\ P_1\ (\lambda f_1{:}\Phi_1.\ (\lambda f_2{:}\Phi_2.\ P_2)))$$

LF variables are used

- to name derived formulas and clauses (as assumptions)
- to introduce new variables of the logic
- to store contextual information

CNF Conversion w/ Partial Clauses

- Tseitin rules will apply elimination and renaming at the same time (no choice of one)
- *Partial Clauses* represent intermediate steps

$$[\![(\phi_1, \cdots, \phi_n; l_1, \cdots, l_n)]\!] \quad = \quad \phi_1 \vee \ldots \vee \phi_n \vee l_1 \vee \ldots \vee l_n$$

- Starts with a single partial clause $(\phi; \cdot)$

$$\begin{array}{lcl} (\phi_1 \wedge \phi_2, \overline{\phi}; C), \Pi & \Rightarrow & (\phi_1, \overline{\phi}; C), (\phi_1, \overline{\phi}; C), \Pi \\ (\phi_1 \vee \phi_2, \overline{\phi}; C), \Pi & \Rightarrow & (\phi_1, \phi_2, \overline{\phi}; C), \Pi \end{array}$$

$$(\phi, \overline{\phi}; C), \Pi \quad \Rightarrow \quad (\overline{\phi}; v, C), \Pi \quad (v \mapsto \phi)$$

$$(\cdot; C), \Pi \quad \Rightarrow \quad C, \Pi$$

LFSC

- Based on Edinburgh Logical Framework (LF)
- Meta-logical proof checker
- Logic declared in user signature
  - Clause, Literal, True/False, Lists …
- If a proof type checks, then it is considered valid
- Optimizations
  - Side Condition Compilation
  - Deferred Resolution

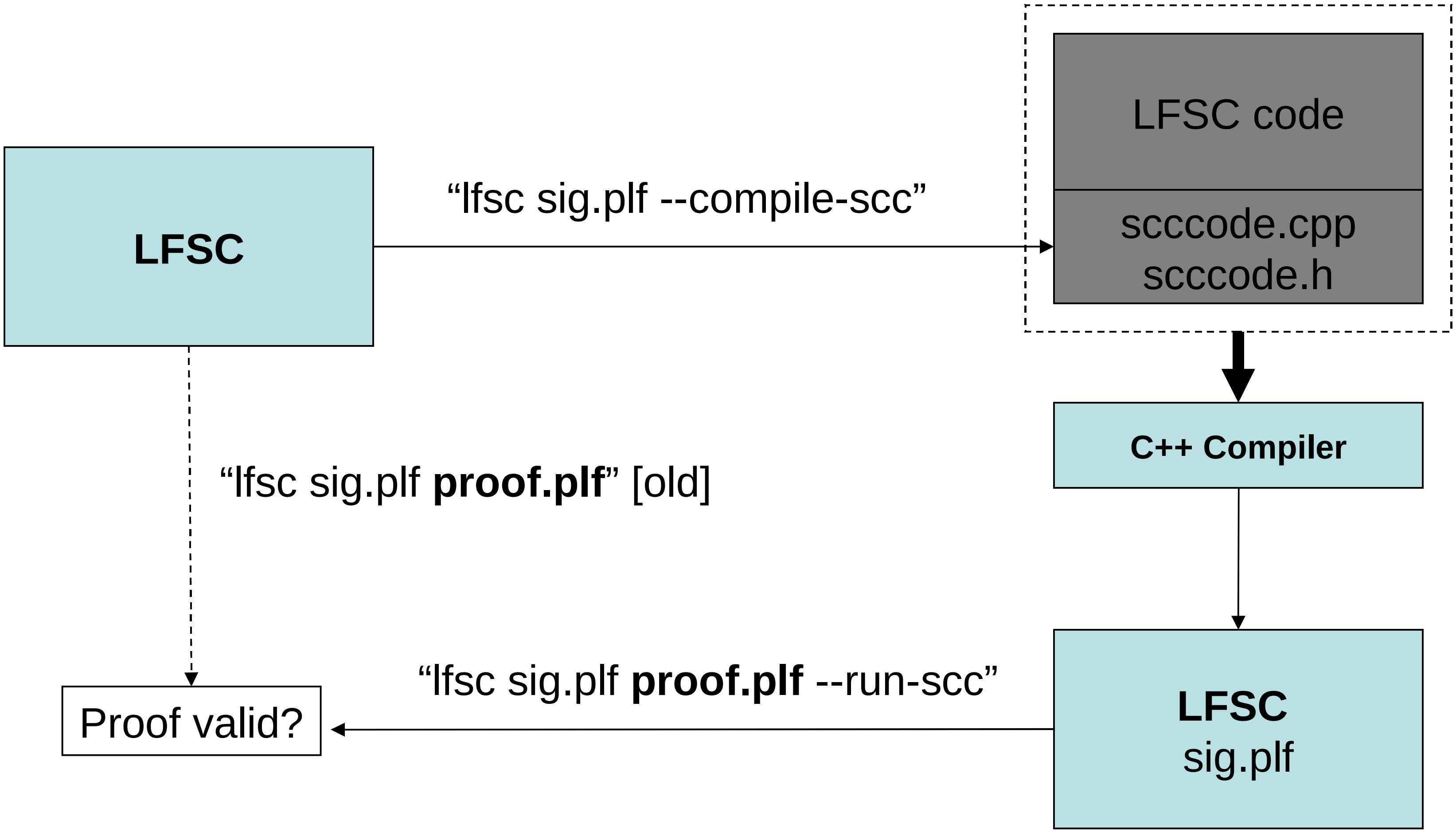
LFSC Side Conditions

- Proofs need computational side conditions
- Example: “resolve” rule for SMT proofs
- Written in simple functional language

```
…
(program append ((c1 clause) (c2 clause)) clause
  (match c1 (cln c2) ((clc l c1') (clc l (append c1' c2)))))
```

- Side conditions executed with interpreter
- Idea: Convert to C++ and execute directly

Approach


LFSC code
scccode.cpp
scccode.h
LFSC
“lfsc sig.plf --compile-scc”
C++ Compiler
“lfsc sig.plf **proof.plf**” [old]
LFSC
sig.plf
“lfsc sig.plf **proof.plf** --run-scc”
Proof valid?

Checking Resolution

- Resolution rule: clauses C and D on variable v
  - C contains v
  - D contains ¬v
  - Removing occurrences of v from C yields C’
  - Removing occurrences of ¬v from D yields D’
  - Appending C’ to D’ yields clause E
  - Duplicate literals eliminated from E
- Naively checked on every resolution step
- Idea: Calculate resolvent clause lazily

Approach

- Extended definition of clauses:
  - cln: empty clause
  - clc L C: clause C with literal L concatenated
  - *clr* L C: clause C with literal L removed
  - *concat* C_1 C_2: append clauses C_1 and C_2 in standard form
- Resolution rule becomes:
  - C contains v
  - D contains $\neg$v
  - Return (concat (clr v C) (clr $\neg$v D))
- Resolution deferred until final step
  - Calculate extended clause
  - Convert extended clause to standard clause

Conversion to Standard Clause

$$[\![G]\!]^{\sigma} = C$$

- Extended clause G, standard clause C, set of literals σ.

...

$$
\begin{aligned}
[\![(clc\ L\ C)]\!]^{\sigma} &= if(L \in \sigma)\ ([\![C]\!]^{\sigma})\ else\ (clc\ L\ [\![C]\!]^{\sigma\ +\ L}) \\
[\![(clr\ L\ C)]\!]^{\sigma} &= [\![C]\!]^{\sigma\ +\ L}
\end{aligned}
$$

- Literals to remove stored in σ
  - Literals marked for deletion eliminated
  - Duplicate literals eliminated

Results

- Benchmarks QF_IDL difficulty 0-3
- Timeout of 1800s
- Public job on SMT EXEC

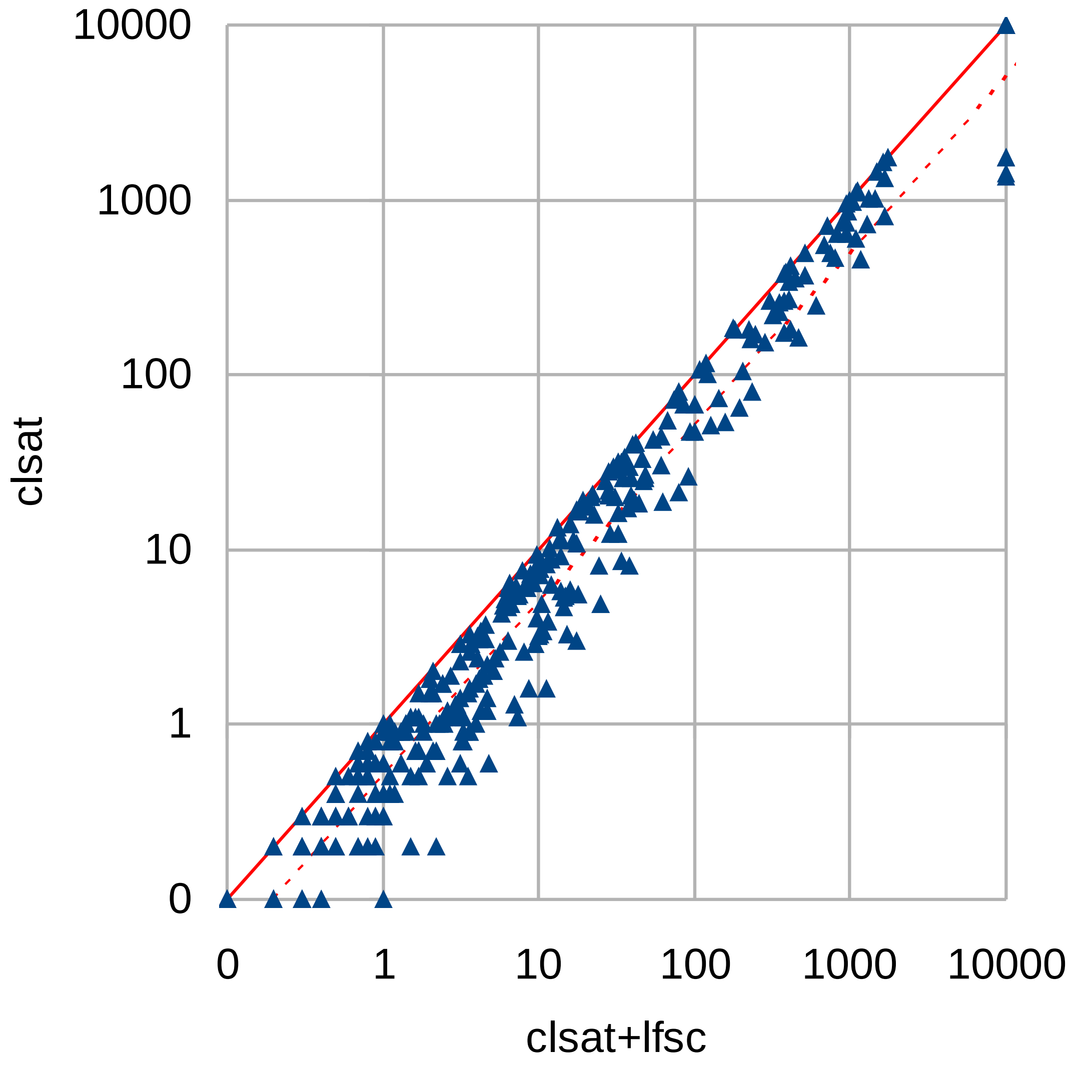
Solver	Score	Unknown	Timeout
clsat (w/o proof)	542/622	50	30
clsat+lfsc (optimized)	538/622	51	33
clsat+lfsc (unoptimized)	485/622	58	79

Results

Solver	Score	Time1	Time2
clsat (w/o proof)	542/622	20168.7s	31843.6s
clsat+lfsc (optimized)	538/622	23741.4s	41420.8s
clsat+lfsc (unoptimized)	485/622	52373.8s	n/a

- Time1: Total time to solve 485 benchmarks solved by all three configurations
- Time2: Total time to solve 538 benchmarks solved by first two configurations

Results



Conclusion

- Provides fast and flexible proof checking
- Proof production overhead is less than 10%
- Lowest reported proof checking time
- Proof checking overhead converging to 2x
  - 30.1% on average