Fast and Flexible Proof Checking with LFSC

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Overview

Logical **F**ramework with **S**ide **C**onditions as:

1. Framework for defining SMT proof systems
2. Optimized Proof Checker
3. Proof System for Linear Real Arithmetic
4. Interpolant Generator via Type Inference

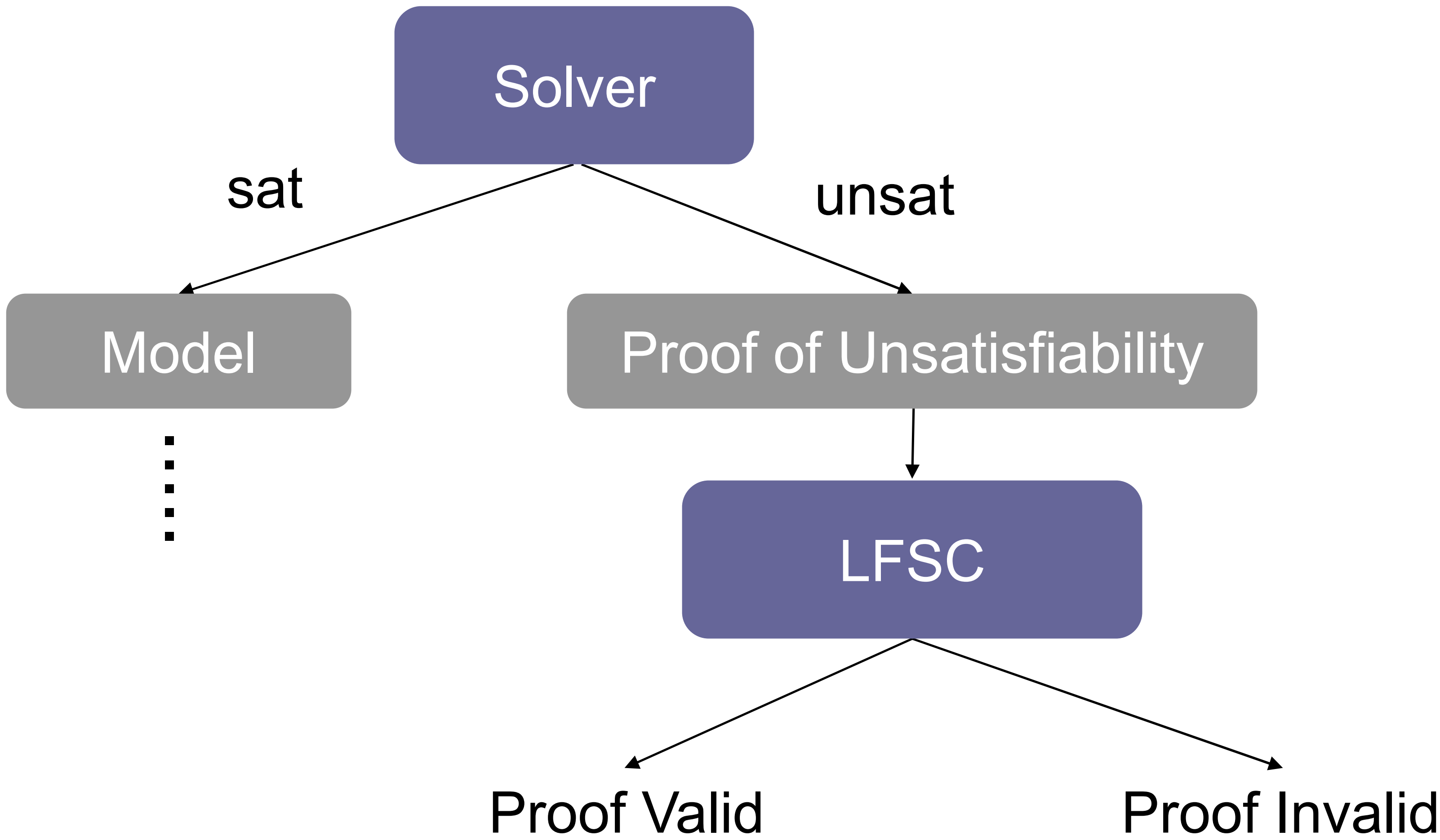
Proof Checking: Motivation

- SMT solvers are difficult to verify
  - Code may be complex (10k+ loc)
  - Code is subject to change

Alternatively....

- Solvers can justify answers with proofs
- There is need for third party certification
  - Must ensure that proof is valid

Proof Checking: Overview



Proof Checking: Challenges

- **Speed**
  - Practical for use with solvers
  - Measured time against solving time
- **Flexibility**
  - Different solvers have different needs
  - Solvers can change over time
  - Many different theories

Proof Checking in LFSC

- Edinburgh Logical Framework (LF) [Harper et al 1993]
  - Based on type theory
  - Meta framework for defining logical systems
- LF with side conditions (LFSC) [Stump et al 2008]
  - Meta-logical proof checker
  - Side Conditions
  - Support for Integer, Rational arithmetic
  - If proof term type-checks,
    Then proof is considered valid

Example proof rule

$$\frac{\psi_1 \quad \psi_2}{\psi_1 \wedge \psi_2}$$

```
(declare and_intro
    (! f1 formula
    (! f2 formula
    (! p1 (proof f1)
    (! p2 (proof f2)
         (proof (and f1 f2)))))))
```

Proof rule with side condition

$$\frac{p > 0}{\bot} \quad \{p \downarrow c,\ c \not> 0\}$$

```
(declare ineq_contradiction
  (! p poly
  (! p1 (proof (> p 0))
  (! s (^ (is_positive (simplify p)) ff)
     (proof false)))))
```

Proof rule with side condition

$$\frac{p > 0}{\bot} \quad \{p \downarrow c,\ c \not> 0\}$$

- Side conditions
  - Written in simply typed functional language

```
simplify ((p poly)) real
  (match p
    ((poly c' l')
      (match (is_zero l')
        (tt c')
        (ff fail)))))
```

Why side conditions?

- Mirror high-performance solver inferences
- More Efficient
  - Smaller Proof Size
  - Faster Checking time
- Amount can be fine tuned

Fully Declarative ⟷ Fully Computational

Proof Systems for LRA

- LFSC for arithmetic [Reynolds et al 10]
- Proofs in Linear Real Arithmetic (LRA)
  - Rules require computational side conditions
    - e.g. $(t_1 + (t_2 + t_3)) = ((t_3 + t_1) + t_2)$
  - Use of side conditions for normalization
    - e.g. $(t_1 + (t_2 + t_3)) \downarrow p_1$, $((t_3 + t_1) + t_2) \downarrow p_2$
    - Verify $p_1 = p_2$ using side conditions

Proof Systems for LRA

- Use SMT solver CVC3 to generate proofs
  - Module to convert proofs to LFSC format
- Flexibility: Multiple Signatures for LRA
  - Declarative
    - Rewrite calculus, native format used by CVC3
    - Rules of form $\Psi_1 \leftrightarrow \Psi_2$
  - Computational
    - Take advantage of LFSC side condition features
    - Rules involving polynomial atoms

Compaction from CVC3 to LFSC

- Theory lemmas in QF_LRA
  - Ex: $\neg(2x>2y) \vee \neg(y>x+5)$
  - Can be done by finding set of coefficients

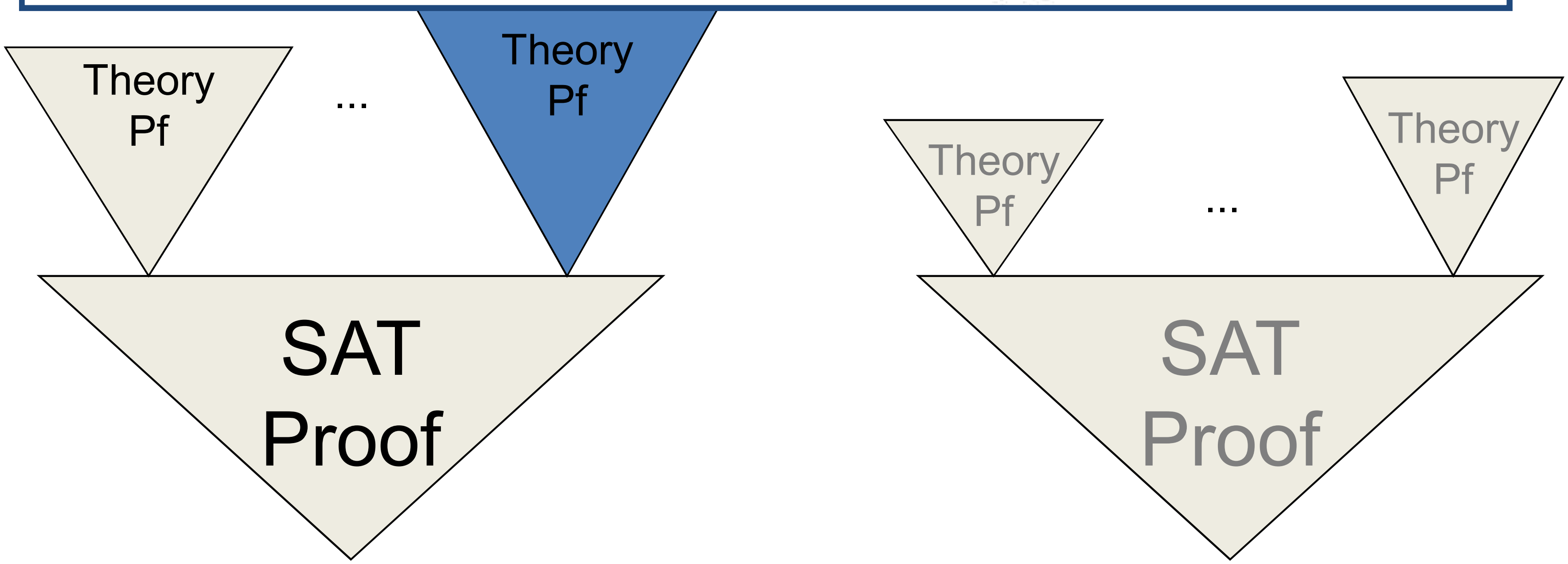
$$\frac{\begin{array}{l} \tfrac{1}{2} * \; 2x > 2y \\ 1 * \; y > x + 5 \end{array}}{x + y > y + x + 5}$$

$$\Downarrow$$

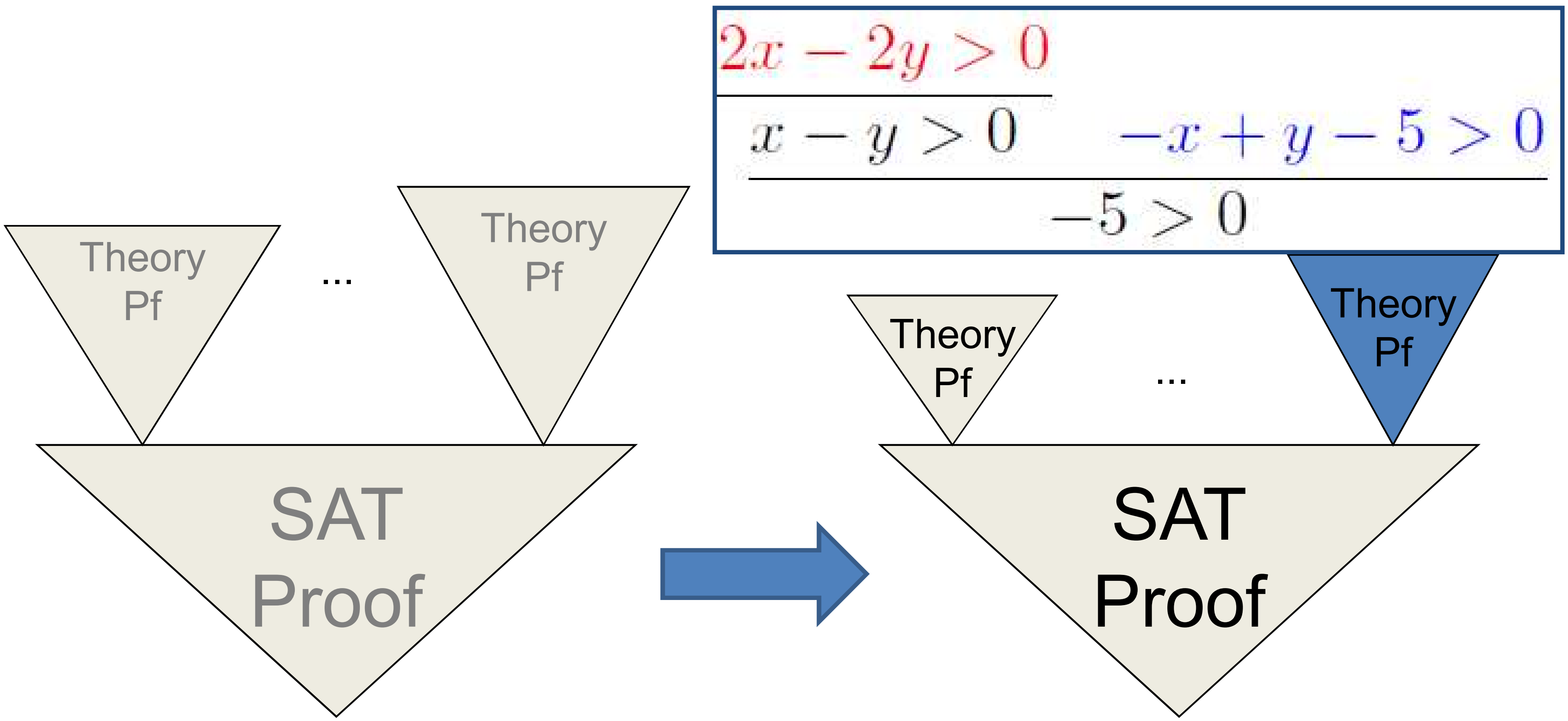
$$0 > 5$$

Proof transformation

$$\dfrac{\dfrac{\dfrac{2x > 2y \quad \dfrac{}{2x > 2y \Leftrightarrow x > y}}{x > y} \quad y > x + 5}{x > x + 5} \quad \dfrac{\vdots}{x > x + 5 \Leftrightarrow \bot}}{\bot}$$

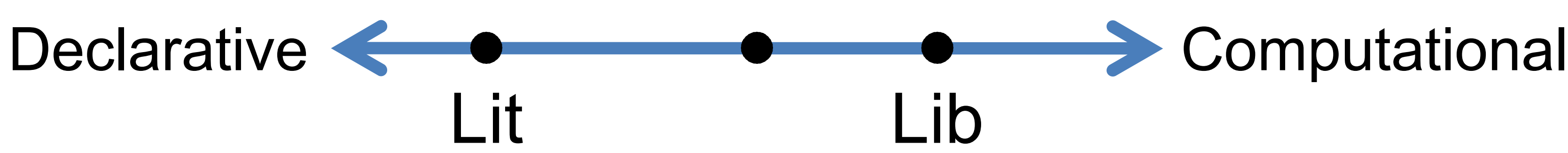


Proof transformation



Experimental results

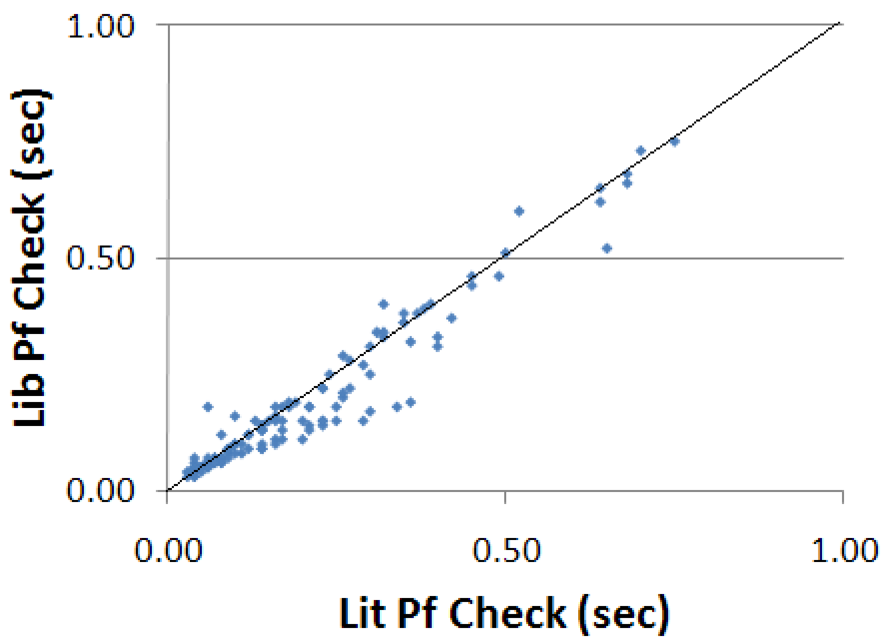
- Configurations
  - Literal translation (**Lit**)
    - Faithful encoding of CVC3's native format
  - Liberal translation (**Lib**)
    - Capitalize on side conditions



Proof checking time

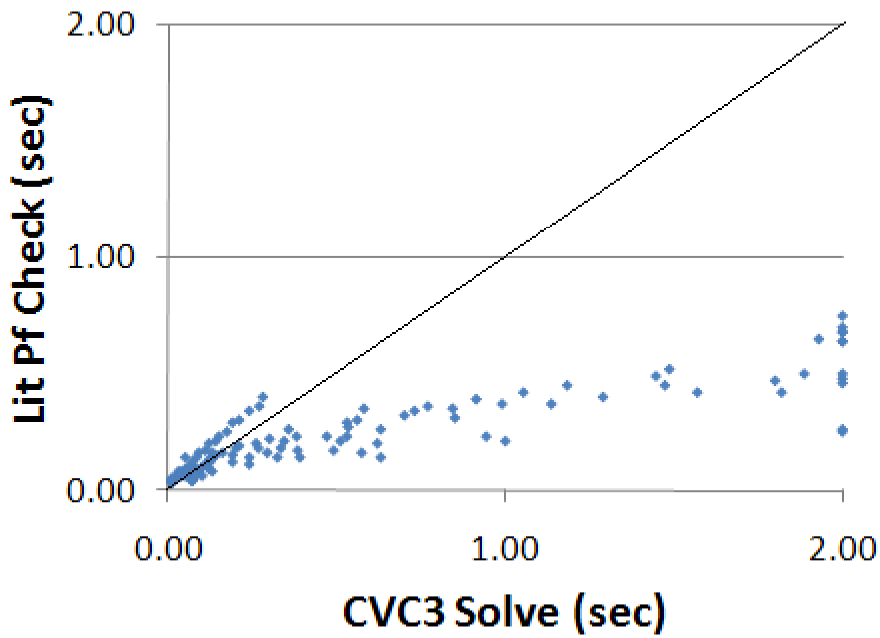
- For theory lemmas: 3x faster proof checking time
  - Theory lemma proofs 5x smaller in size on average

Lit vs Lib

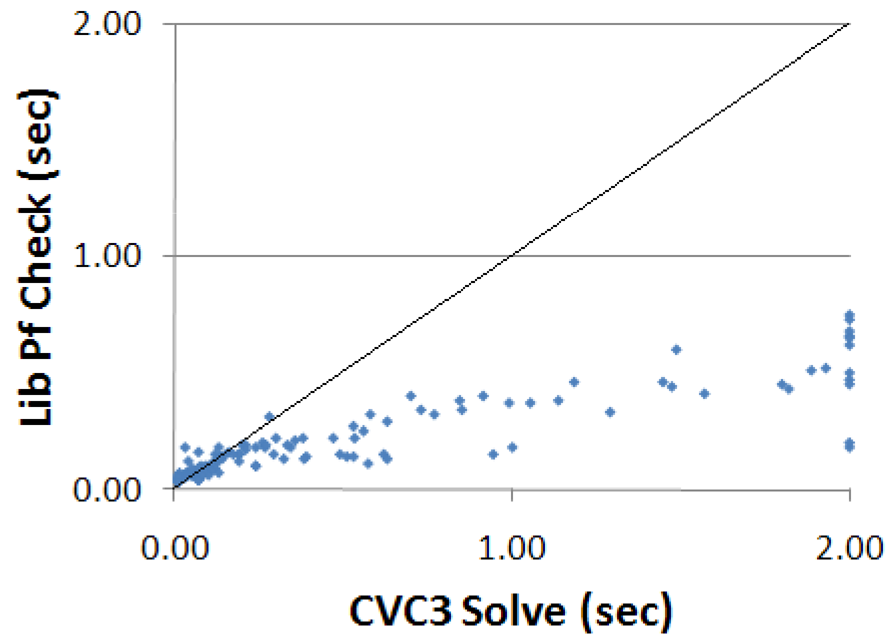


Proof checking vs Solving

Solving vs Lit



Solving vs Lib



- Proof checking ~10x faster than solving

Interpolation

- In addition to proofs of unsatisfiability, use LFSC for richer proof calculi

- Interpolant generating proofs [Reynolds et al 11]
  - Theory of uninterpretted functions with equality (EUF)

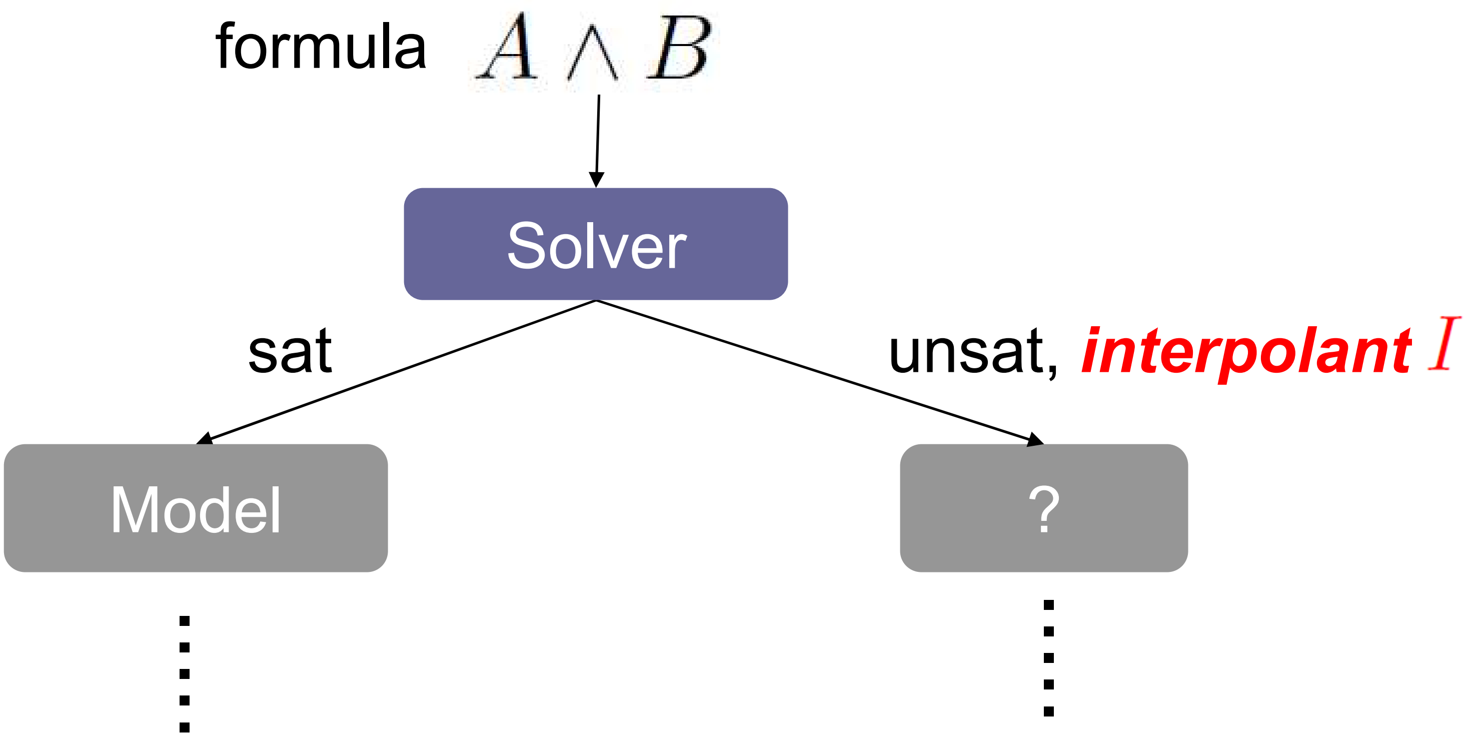
Interpolation

- For theory T, a T-interpolant I for (A,B)

$$(1)\ A \models_T I$$
$$(2)\ B, I \models_T \bot$$
$$(3)\ L(I) \subseteq L(A) \cap L(B)$$

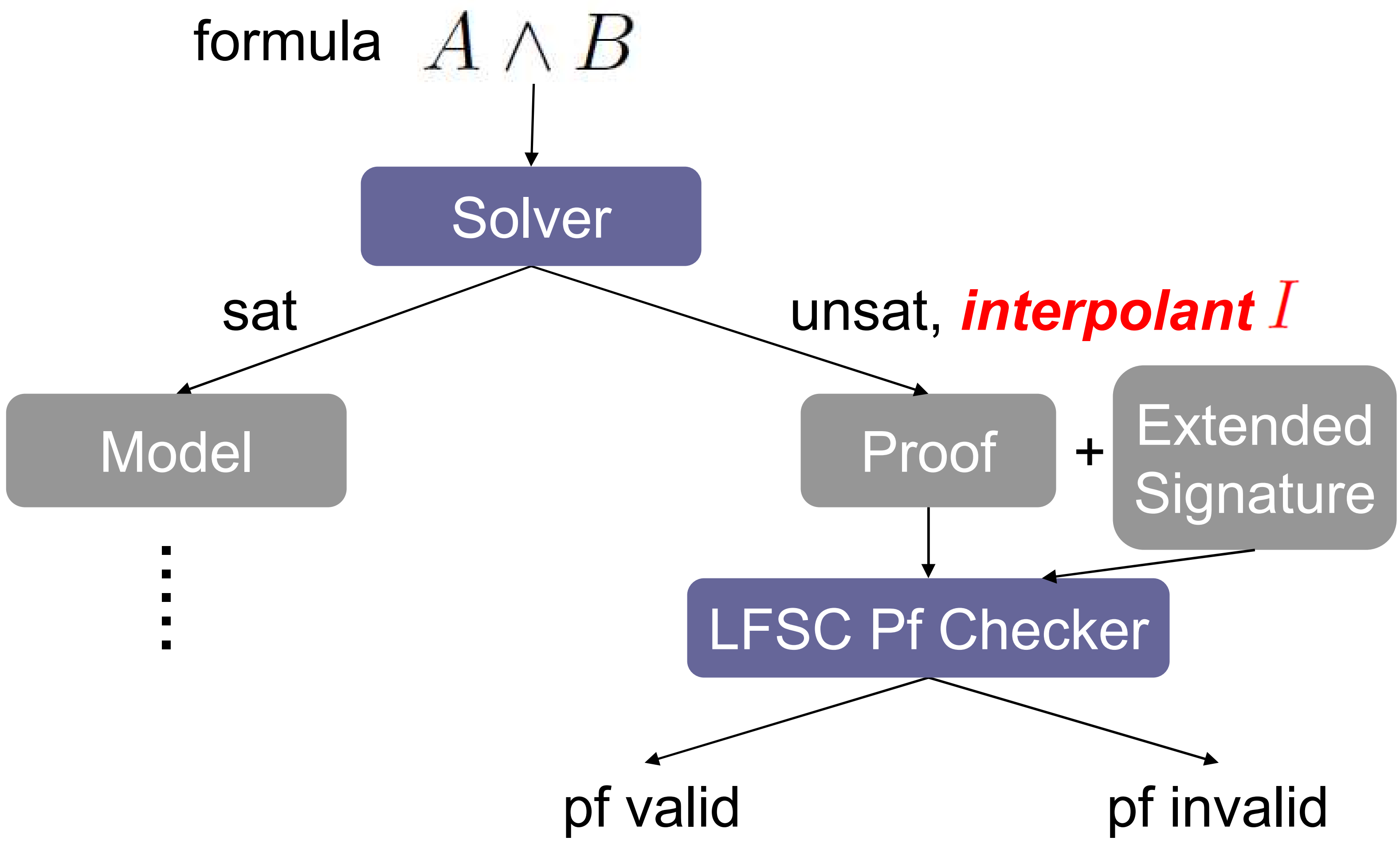
- In some cases, may be efficiently generated from proofs
- Applications
  - Model Checking, Predicate Abstraction, ...
- Use LFSC to generate *certified* interpolants

Proof Checking: Interpolants



- Since LFSC is meta-framework, we can extend signature to type-check proofs about interpolants

Proof Checking: Interpolants



Proof Checking: Interpolants

```
(check
      (% ...
      (% a (proof A)
      (% b (proof B)
      (: (interpolant A B I )
           P
         ))...)
```

- Check if P is of type **(`interpolant`** *A B I***)**, for formulas *A*, *B*, *I*
- If so, then *I* is a certified interpolant for (*A*, *B*)

Proof Checking: Interpolants

- SMT solver produces interpolant + proof

- LFSC verifies that proof:

  (1) Successfully type checks, and

  (2) Shows claimed interpolant is an interpolant.

- Solver + Checker must agree on the interpolant

Interpolant Generation via Proof Checking

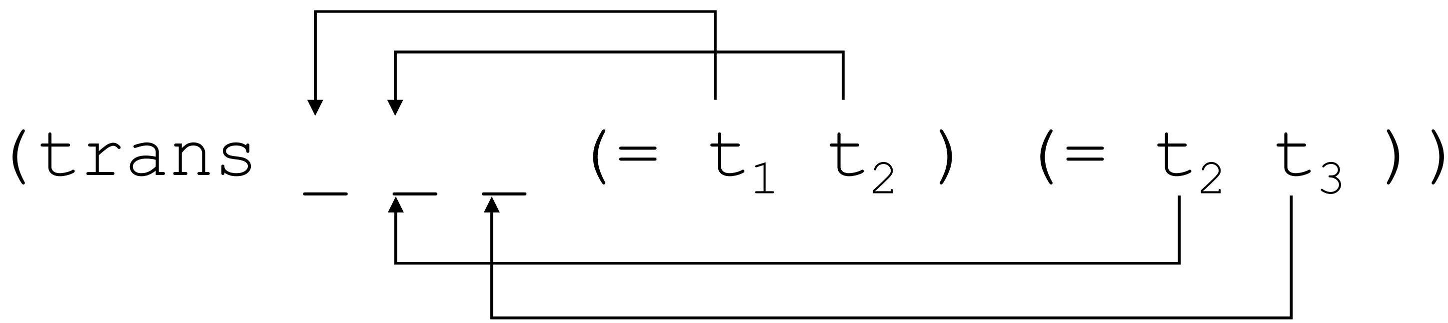
- Alternatively:

  Use proof checker as the interpolant generator

- Solver writes proof in same signature
  - Constructs proof of type `(interpolant` $A\ B\ I$`)`,
    - for some value of I, unknown a priori
  - Value of I computed by type inference

Interpolant Generation via Proof Checking

- LFSC proofs may contain hole symbols "_"
- For example:



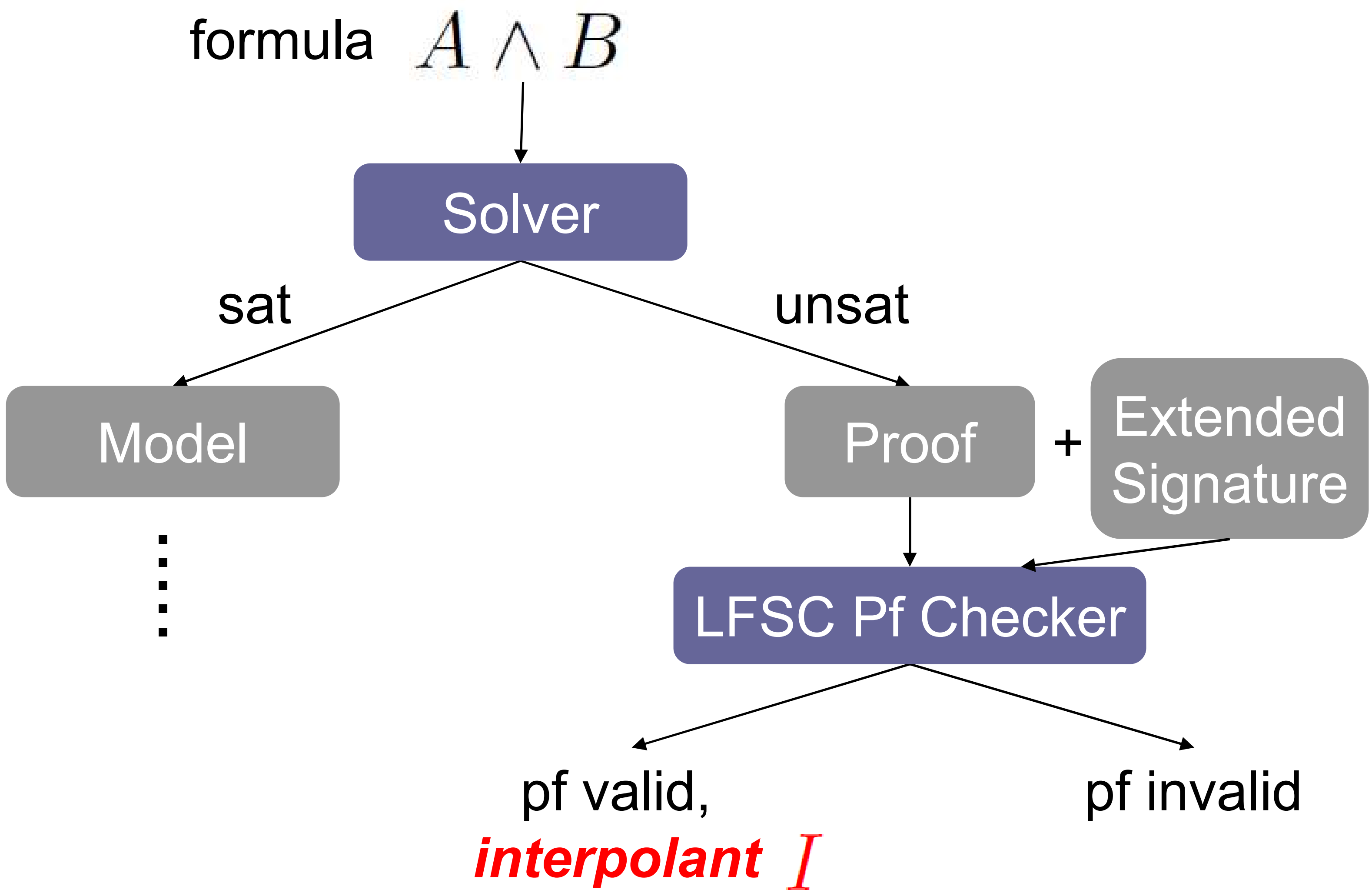
- Allow proof checker fill in value for interpolant
  - Certified correct by construction

Interpolant Generation via Proof Checking

```
(check
      (% ...
      (% a (proof A)
      (% b (proof B)
      (: (interpolant A B _)
           P
         ))...)
```

- The interpolant field left unspecified "_"
- If P is of type **(interpolant** $A\ B\ I$**)**,
  - Value of I is given to user
  - I is a certified interpolant for (A, B)

Interpolant Generation via Proof Checking



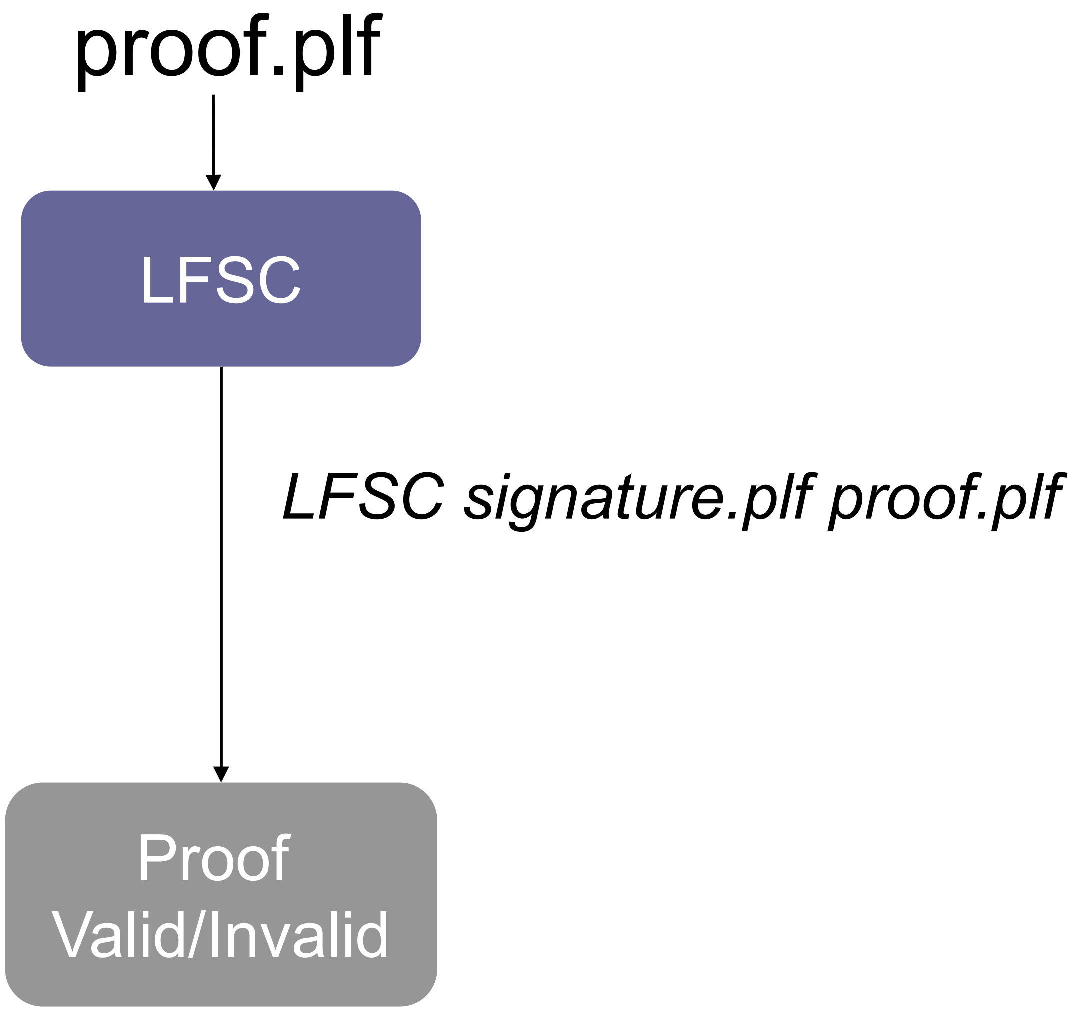
Results

- Tested configurations
  - **euf**: proof checking
  - **eufi**: proof checking with interpolant generation
- Proof checking fast w.r.t to solving
  - **euf** 11x faster than solving
  - **eufi** 5x faster than solving
- Interpolants come at small overhead
  - **eufi** 22% overhead with respect to solving + pf generation

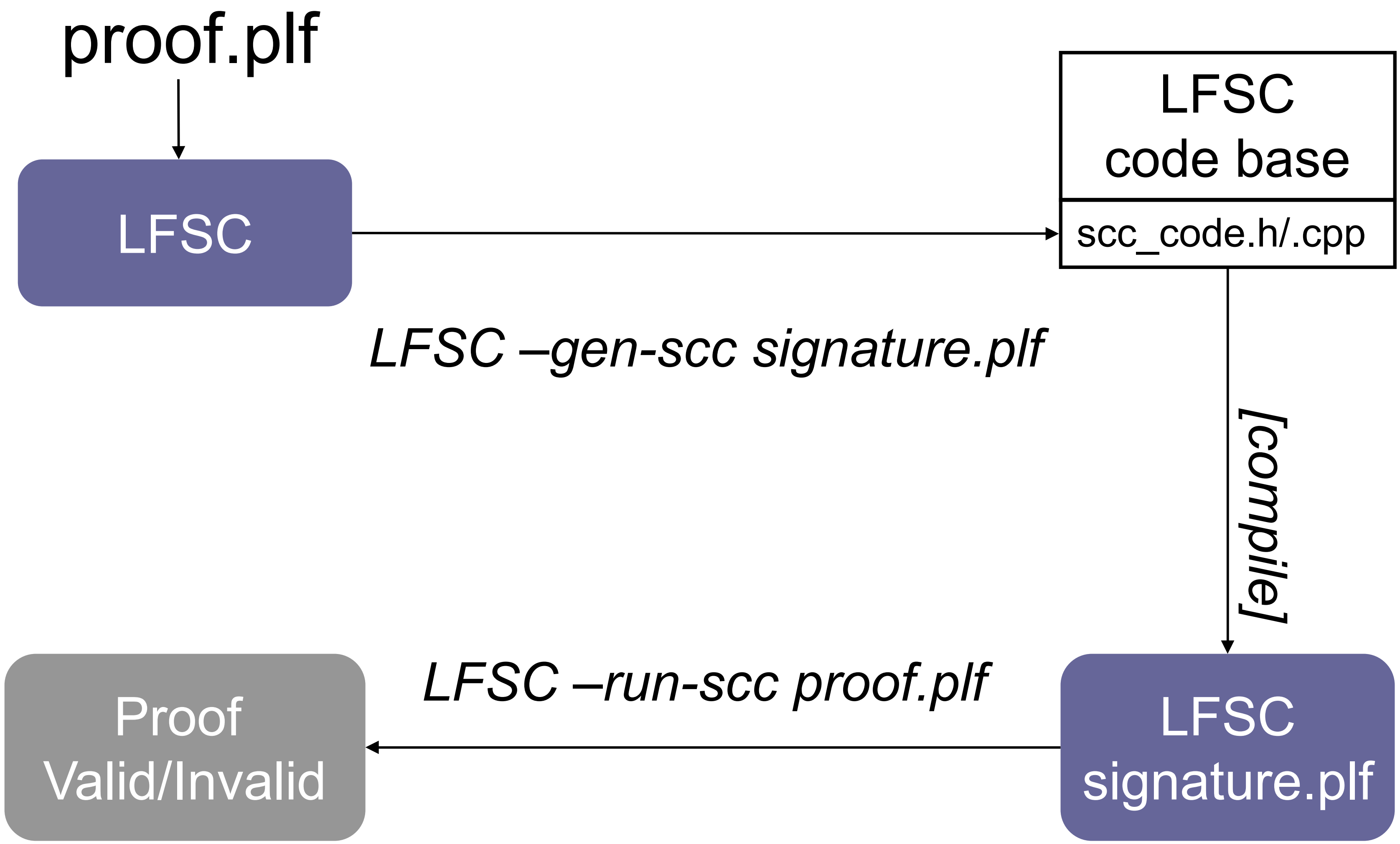
Other Optimizations

- Optimizations for LFSC
  - Incremental Checking
    - Proof is checked while it is parsed
      - Instead of being read into memory
  - Optimized boolean resolution checking
    - Resolvent clauses produced lazily
  - Signature Compilation [Oe et al 09]
    - Side conditions run directly in compiled C++
      - Instead of using an interpreter

Signature Compilation



Signature Compilation



Future Work

- Integration into CVC4
  - Extensions to other theories
    - Datatypes, Bit Vectors, Arrays, etc.
- New release of LFSC
  - Usability of user-defined signatures
  - Improved performance
  - …