A Decision Procedure for String to Code Point Conversion

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SMT Solvers for Strings

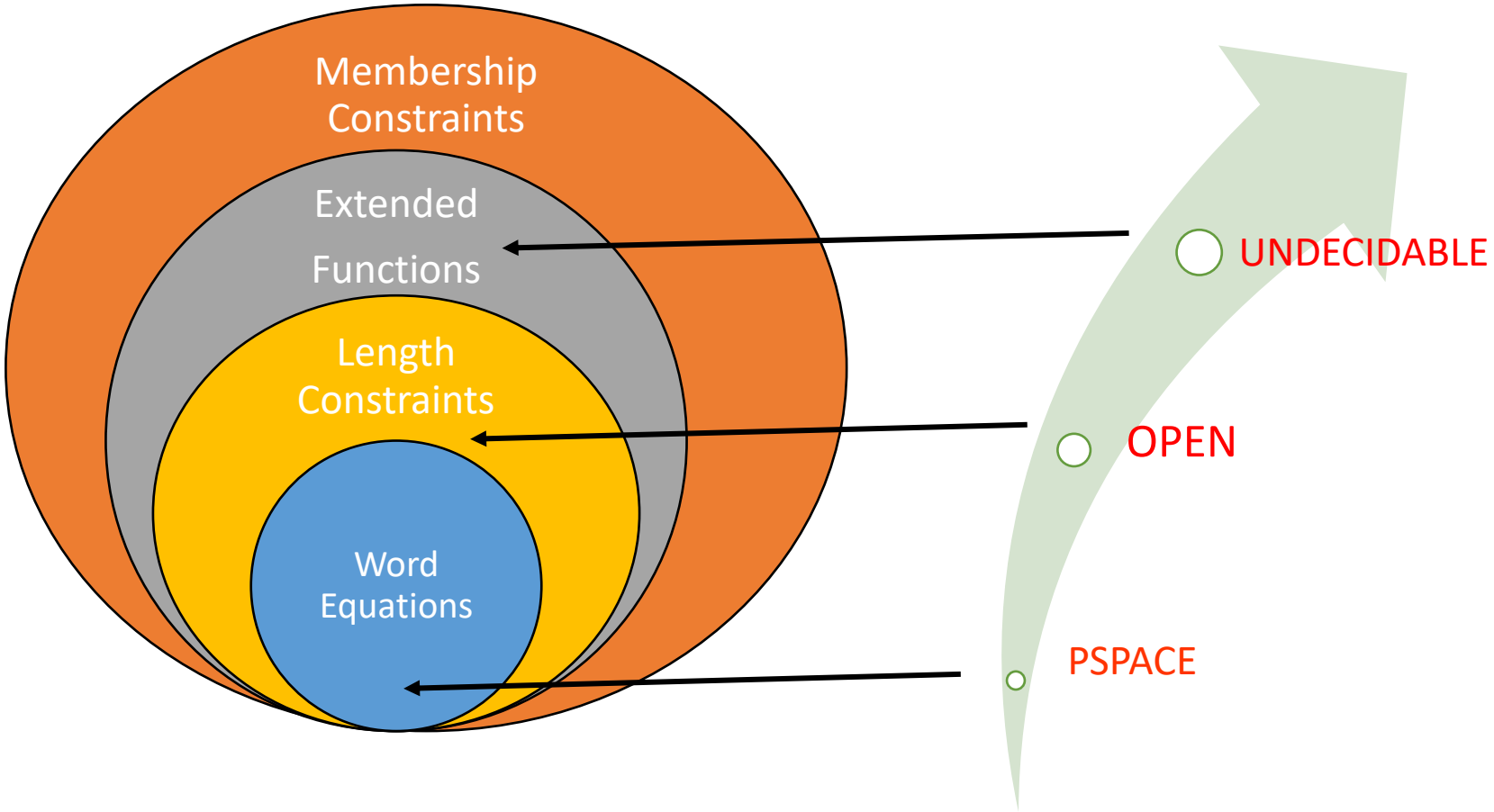
- Strings are important in formal methods applications:
  - SQL injections, XSS vulnerabilities in web applications
  - Reasoning about access policies in cloud applications
  - Data processing

- In the past decade, efficient SMT solvers for strings have emerged
  - **cvc4**, Norn, S3 , Trau, Z3, Z3str, …

Overview

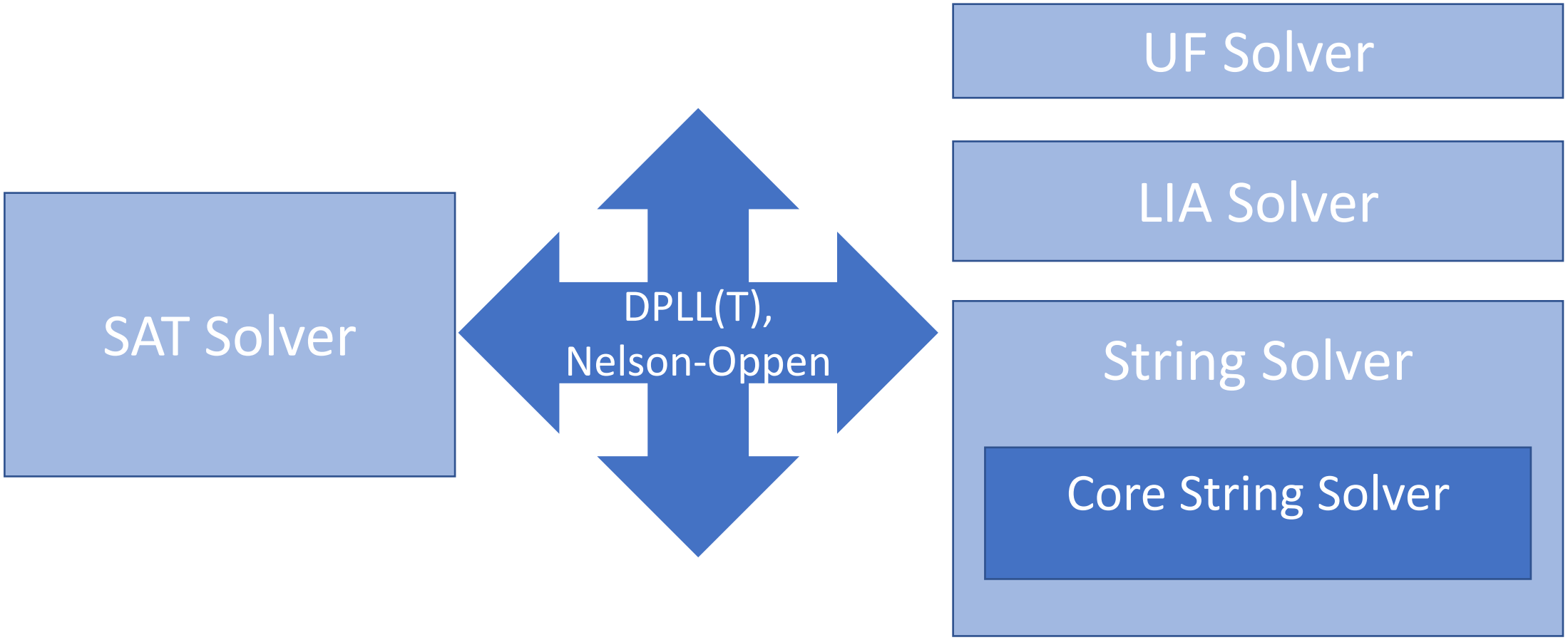
- A decision procedure for a simple theory of strings T_{AS}
  - Strings + length constraints + *string-to-code conversion*
- Improved reductions for extended string functions
- Implementation in CVC4
- Experimental Results
  - Significant improvement in state-of-the-art for SMT string solvers

Strings and RegExp: Theoretical Challenges



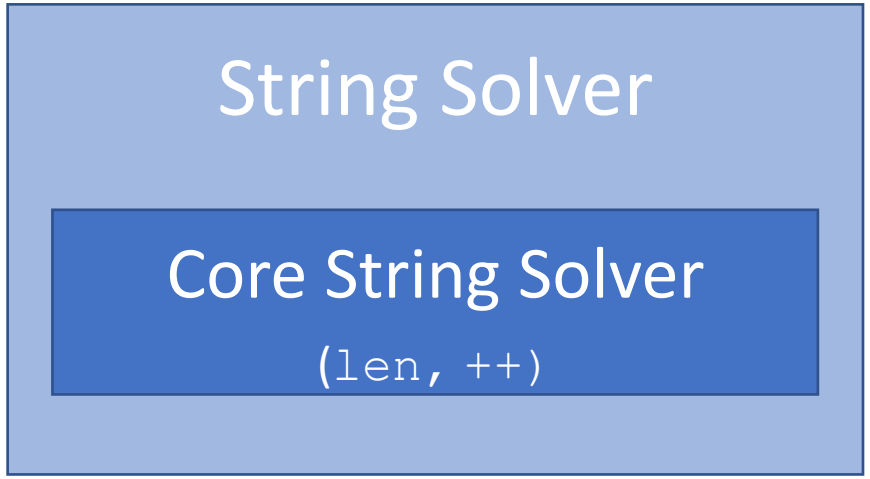
- Many applications require *extended string functions* and *RegEx memberships*
  - `to_int(x)≠44, to_lower(x)="abc", x∈range("A","Z")`

SMT Solvers for Strings + Extended Functions



SMT Solvers for Strings + Extended Functions

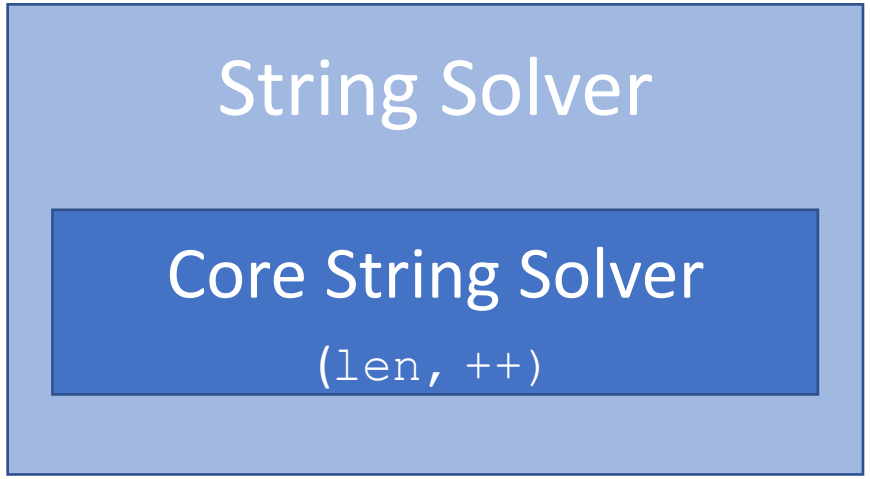
- *Core* string solver **[Liang et al CAV 2014]**
  - (Dis)equalities over strings:
    - Variables, constants, concatenation `++`
  - (Dis)equalities with string length `len`



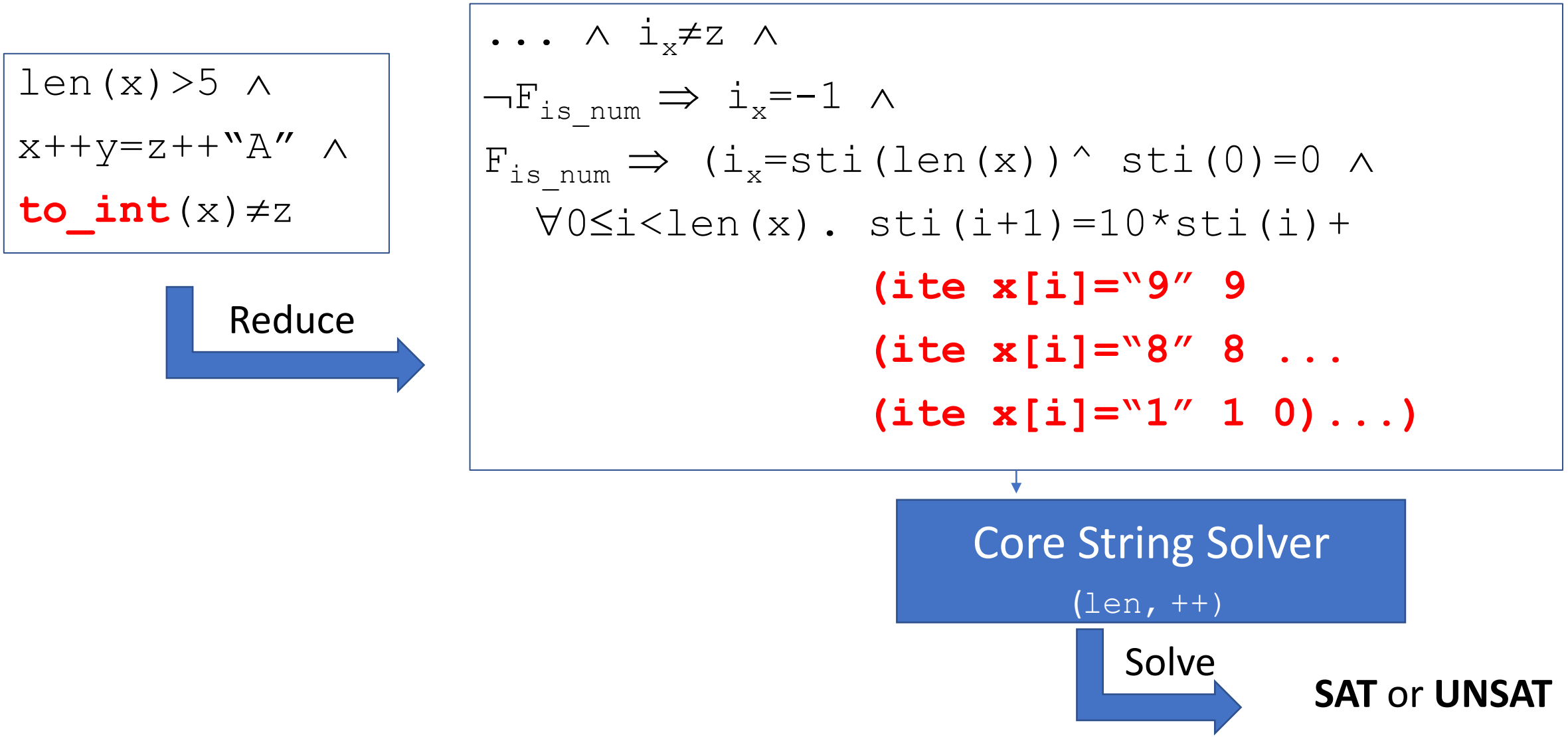
SMT Solvers for Strings + Extended Functions

```
len(x)>5 ∧
x++y=z++"A" ∧
to_int(x)≠z
```

- Extended functions can be reduced to word equations
  [Reynolds et al CAV 2017]



SMT Solvers for Strings + Extended Functions



SMT Solvers for Strings + Extended Functions (via code)

$\texttt{len(x)>5} \wedge$
$\texttt{x++y=z++“A”} \wedge$
$\texttt{to\_int(x)}\neq\texttt{z}$

Reduce

$\ldots \wedge i_x \neq z \wedge$
$\neg F_{is\_num} \Rightarrow i_x = -1 \wedge$
$F_{is\_num} \Rightarrow (i_x = sti(len(x)) \wedge sti(0)=0 \wedge$
$\forall 0 \leq i < len(x).\ sti(i+1) = 10*sti(i) +$
(code(x[i])-48)

- **In this paper:**
  - New procedure for string-to-code point
  - Enables efficient reductions for constraints in common applications

Core String Solver
(len, ++, ***code***)

Solve

SAT or **UNSAT**

Signature of Theory of Strings Σ_{AS}

Sorts

```
Int        Str        Bool
```

Σ_A

```
n  :  Int                          for all n∈Z
+  :  Int x Int  →  Int
-  :  Int  →  Int
≥  :  Int x Int  →  Bool
```

Σ_S

```
l     :  Str                       for all l∈𝒜*
len   :  Str  →  Int
code  :  Str  →  Int
```

… (Dis)equalities $=, \neq$ over all sorts

$\Rightarrow$ Intentionally omit string concatenation

String to code operator code

- Assume ordering on characters of alphabet $\mathcal{A}$:
  - $c_1 < \ldots < c_{|\mathcal{A}|-1}$
  - For each c_i, we call i its code point
- $\texttt{code} : \texttt{Str} \rightarrow \texttt{Int}$ is interpreted as:
  1. For w in $\mathcal{A}^1$, $\texttt{code}(w)$ is the code point of the single character in w
  2. For all other w, $\texttt{code}(w)$ is -1

- Notes:
  - In practice, we are interested in reasoning about Unicode strings
    - E.g. $\texttt{code}(\text{“A”}) = 65$, $\texttt{code}(\text{“5”}) = 53$
  - Alphabet can have any cardinality (≥ 1)

Rule-based calculus for strings

- Rules operate on configurations <A, S> where:
  - **A:** a set of arithmetic constraints
  - **S:** a set of string constraints
- Each rule in guarded assignment form:

$$\textbf{RuleName}\ \frac{\text{required conditions}}{\text{modifications to the configuration}} \quad \text{or} \quad \frac{\text{required conditions}}{\textbf{“unsat”}}$$

- Some rules have multiple conclusions, delimited by “||”:

$$\frac{\text{required conditions}}{\text{modification \#1} \quad || \quad \ldots \quad || \quad \text{modification \#n}}$$

Rule-based calculus for strings

- A run of the procedure generates a ***derivation tree***:
  - Nodes are configurations
  - Child nodes are obtained by legally applying a rule to its parent

- Given derivation tree with root <A, S>
  - A closed derivation tree (where all leaves are "**unsat**")

    $\Rightarrow$ <A, S> is T_{AS}-unsat
  - A derivation tree with a saturated configuration

    $\Rightarrow$ <A, S> is T_{AS}-sat

Core rules

$$\text{A-Conf}\ \frac{\mathsf{A} \models_{\mathsf{LIA}} \bot}{\text{unsat}} \qquad \text{A-Prop}\ \frac{\mathsf{A} \models_{\mathsf{LIA}} s \approx t \quad s,t \in \mathcal{T}(\mathsf{A} \cup \mathsf{S})}{\mathsf{S} := \mathsf{S}, s \approx t}$$

$$\text{S-Conf}\ \frac{\mathsf{S} \models \bot}{\text{unsat}} \qquad \text{S-Prop}\ \frac{\mathsf{S} \models s \approx t \quad s,t : \mathsf{Int} \quad s,t \in \mathcal{T}(\mathsf{A} \cup \mathsf{S})}{\mathsf{A} := \mathsf{A}, s \approx t}$$

$$\text{L-Intro}\ \frac{\mathsf{S} \models x \approx l \quad x : \mathsf{Str}}{\mathsf{S} := \mathsf{S},\ \mathsf{len}(x) \approx (\mathsf{len}(l))\downarrow} \qquad \text{L-Valid}\ \frac{x \in \mathcal{T}(\mathsf{A} \cup \mathsf{S}) \quad x : \mathsf{Str}}{\mathsf{S} := \mathsf{S}, x \approx \epsilon \quad \| \quad \mathsf{A} := \mathsf{A},\ \mathsf{len}(x) > 0}$$

$$\text{Card}\ \frac{\mathsf{S} \models \mathsf{len}(x_1) \approx \ldots \approx \mathsf{len}(x_n) \quad n > 1}{\|_{1 \leqslant i < j \leqslant n}\ \mathsf{S} := \mathsf{S}, x_i \approx x_j \quad \| \quad \mathsf{A} := \mathsf{A}, \mathsf{len}(x_1) > \lfloor \log_{|\mathcal{A}|}(n-1) \rfloor}$$

- Rules adopted from previous work **[Liang et al CAV 2014]**

Core rules

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Models LIA solver :

- Conflicts
- Propagate Int equalities from A to S

$$\text{S-Conf}\ \frac{\mathsf{S} \models \bot}{\text{unsat}} \qquad \text{S-Prop}\ \frac{\mathsf{S} \models s \approx t \quad s,t : \mathsf{Int} \quad s,t \in \mathcal{T}(\mathsf{A} \cup \mathsf{S})}{\mathsf{A} := \mathsf{A}, s \approx t}$$

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Core rules

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Models core string solver :

- Conflicts by pure equality reasoning
- Propagate Int equalities from S to A

$$\text{L-Intro}\ \frac{\mathsf{S} \models x \approx l \quad x : \mathsf{Str}}{\mathsf{S} := \mathsf{S},\ \mathsf{len}(x) \approx (\mathsf{len}(l))\downarrow} \qquad \text{L-Valid}\ \frac{x \in \mathcal{T}(\mathsf{A} \cup \mathsf{S}) \quad x : \mathsf{Str}}{\mathsf{S} := \mathsf{S}, x \approx \epsilon \quad \| \quad \mathsf{A} := \mathsf{A},\ \mathsf{len}(x) > 0}$$

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Core rules

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Reasoning about length:

- Fixed for constant strings
- Length is non-negative

- Rules adopted from previous work **[Liang et al CAV 2014]**

Core rules

$$\text{A-Conf}\ \frac{\mathsf{A} \models_{\mathsf{LIA}} \bot}{\text{unsat}} \qquad \text{A-Prop}\ \frac{\mathsf{A} \models_{\mathsf{LIA}} s \approx t \quad s,t \in \mathcal{T}(\mathsf{A} \cup \mathsf{S})}{\mathsf{S} := \mathsf{S}, s \approx t}$$

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$$\text{Card}\ \frac{\mathsf{S} \models \mathsf{len}(x_1) \approx \ldots \approx \mathsf{len}(x_n) \quad n > 1}{\|_{1 \leqslant i < j \leqslant n}\ \mathsf{S} := \mathsf{S}, x_i \approx x_j \quad \| \quad \mathsf{A} := \mathsf{A}, \mathsf{len}(x_1) > \lfloor \log_{|\mathcal{A}|}(n-1) \rfloor}$$

Reasoning about cardinality :

- Only finitely many strings exist for each length, since alphabet $\mathcal{A}$ is finite.

- Rules adopted from previous work **[Liang et al CAV 2014]**

Rules for code points (1/3)

$$\text{C-Intro}\ \frac{\mathsf{S} \models x \approx l \quad l \in \mathcal{A}^1}{\mathsf{A} := \mathsf{A}, \mathsf{code}(x) \approx (\mathsf{code}(l))\downarrow} \qquad \text{C-Collapse}\ \frac{\mathsf{S} \models x \approx l \quad \mathsf{code}(x) \in \mathcal{T}(\mathsf{S})}{\mathsf{A} := \mathsf{A}, \mathsf{code}(x) \approx (\mathsf{code}(l))\downarrow}$$

- Code of string `x` must be equal to its value, if it is forced by an equality
  - If x is equal to a constant string of length one:
    - `x="A"∈ S` … `code(x)=65 ∈ A`
  - If x is equal to any constant string, and code(x) is a term in S
    - `x="abc", code(x)=code(y)∈ S` … `code(x)=-1 ∈ A`

Rules for code points (2/3)

$$\text{C-Valid}\ \frac{\mathsf{code}(x) \in \mathcal{T}(\mathsf{S})}{\mathsf{A} := \mathsf{A}, \mathsf{len}(x) \not\approx 1, \mathsf{code}(x) \approx -1 \quad \| \quad \mathsf{A} := \mathsf{A}, \mathsf{len}(x) \approx 1, 0 \leqslant \mathsf{code}(x) < |\mathcal{A}|}$$

- For each string $\mathtt{x}$:
  - The length of $\mathtt{x}$ is not $\mathtt{1}$ and its code point is $-\mathtt{1}$
  - The length of $\mathtt{x}$ is $\mathtt{1}$ and its code point is in range $[\mathtt{0}\ldots|\mathcal{A}|-\mathtt{1}]$

Rules for code points (3/3)

$$\text{C-Inj}\ \frac{\mathsf{code}(x), \mathsf{code}(y) \in \mathcal{T}(\mathsf{S}) \quad x, y \text{ distinct}}{\mathsf{A} := \mathsf{A}, \mathsf{code}(x) \approx -1 \quad \| \quad \mathsf{A} := \mathsf{A}, \mathsf{code}(x) \not\approx \mathsf{code}(y) \quad \| \quad \mathsf{S} := \mathsf{S}, x \approx y}$$

- Code is injective over the domain of strings of length 1
  - For each `code(x)`, `code(y)`, either:
  1. One of these strings, w.l.o.g. assume `x`, is not length `1` … `code(x)=-1`
  2. `x` and `y` have different code points … `code(x)≠code(y)`
  3. `x` and `y` are the same string … `x=y`

Example #1

$\langle A_0, S_0 \rangle$

- A = {`len(x)>len(y), code(x)=code(y), code(x)≥0`}
- S = {}

Example #1

$$\frac{\langle A_0, S_0 \rangle}{\mathbf{code(x)=code(y) \in S}}\ \text{A-Prop}$$

- A = {len(x)>len(y), code(x)=code(y), code(x)≥0}
- S = {code(x)=code(y)}

Example #1

$$\dfrac{\dfrac{\langle A_0, S_0\rangle}{\mathbf{code(x)=code(y)\in S}}\ \text{A-Prop}}{\texttt{code(x)=-1}\in\texttt{A} \qquad \texttt{code(x)}\neq\texttt{code(y)}\in\texttt{A} \qquad \texttt{x=y}\in\texttt{S}}\ \text{C-Inj}$$

- A = {$\texttt{len(x)>len(y)}$, $\texttt{code(x)=code(y)}$, $\texttt{code(x)}\geq 0$}
- S = {$\texttt{code(x)=code(y)}$}

Example #1

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\langle A_0, S_0\rangle}}{\texttt{code(x)=code(y)}\in S}\ \text{A-Prop}}{\texttt{code(x)}\neq\texttt{code(y)}\in A}\quad \texttt{x=y}\in S}{\texttt{code(x)=-1}\in A}\ \text{C-Inj}}{\textbf{unsat}}\ \text{A-Conf}$$

- A = {len(x)>len(y), code(x)=code(y), code(x)≥0, code(x)=-1}
- S = {code(x)=code(y)}

Example #1

$$\dfrac{\dfrac{\dfrac{\langle A_0, S_0\rangle}{code(x)=code(y)\in S}\ \text{A-Prop}}{\dfrac{code(x)=-1\in A}{\textbf{unsat}}\ \text{A-Conf}} \quad \dfrac{\mathbf{code(x)\neq code(y)\in A}}{\textbf{unsat}}\ \text{A-Conf} \quad x=y\in S}{}\ \text{C-Inj}$$

- A = {$len(x)>len(y)$, $\underline{code(x)=code(y)}$, $code(x)\geq 0$, $\underline{code(x)\neq code(y)}$}
- S = {$code(x)=code(y)$}

Example #1

$$\dfrac{\dfrac{\dfrac{\langle A_0, S_0\rangle}{\texttt{code(x)=code(y)}\in S}\ \text{A-Prop}}{\texttt{code(x)=-1}\in A}\ \text{A-Conf}}{\textbf{unsat}} \qquad \dfrac{\dfrac{\texttt{code(x)}\neq\texttt{code(y)}\in A}{\textbf{unsat}}\ \text{A-Conf}}{\textbf{unsat}} \qquad \dfrac{}{\mathbf{x=y}\in S}\ \text{C-Inj}$$

- A = {len(x)>len(y), code(x)=code(y), code(x)≥0}
- S = {code(x)=code(y), x=y}

Example #1

$$\dfrac{\dfrac{\dfrac{\langle A_0, S_0\rangle}{\texttt{code(x)=code(y)}\in S}\ \text{A-Prop}}{\texttt{code(x)=-1}\in A}\ \text{A-Conf}\quad \dfrac{\texttt{code(x)}\neq\texttt{code(y)}\in A}{\textbf{unsat}}\ \text{A-Conf}}{\textbf{unsat}}\ \text{C-Inj}\qquad \dfrac{x=y\in S}{\textbf{len(x)=len(y)}\in \textbf{A}}\ \text{S-Prop}$$

- A = {len(x)>len(y), code(x)=code(y), code(x)≥0, len(x)=len(y)}
- S = {code(x)=code(y), x=y}

Example #1

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\langle A_0, S_0\rangle}}{\texttt{code(x)=code(y)}\in S}\,\text{A-Prop}}{\texttt{code(x)=-1}\in A}\,\text{A-Conf}}{\textbf{unsat}} \quad \dfrac{\dfrac{\texttt{code(x)}\neq\texttt{code(y)}\in A}{}\,\text{A-Conf}}{\textbf{unsat}} \quad \dfrac{\dfrac{\dfrac{}{\texttt{x=y}\in S}\,\text{S-Prop}}{\texttt{len(x)=len(y)}\in A}\,\text{A-Conf}}{\textbf{unsat}}}{}\,\text{C-Inj}$$

- A = {$\underline{\texttt{len(x)>len(y)}}$, $\texttt{code(x)=code(y)}$, $\texttt{code(x)}\geq 0$, $\underline{\texttt{len(x)=len(y)}}$}
- S = {$\texttt{code(x)=code(y)}$, $\texttt{x=y}$}

Example #1

$\langle \mathtt{A}_0, \mathtt{S}_0 \rangle$

A-Prop

$\mathtt{code(x)=code(y)} \in \mathtt{S}$

C-Inj

$\mathtt{code(x)=-1} \in \mathtt{A}$

A-Conf

unsat

$\mathtt{code(x)} \neq \mathtt{code(y)} \in \mathtt{A}$

A-Conf

unsat

$\mathtt{x=y} \in \mathtt{S}$

S-Prop

$\mathtt{len(x)=len(y)} \in \mathtt{A}$

A-Conf

unsat

- Closed derivation tree … $\therefore$ $\mathtt{A}_0 \wedge \mathtt{S}_0$ is T_{AS}-unsatisfiable

Example #2

$\langle A_0, S_0 \rangle$

- A = $\{106 \geq code(x) \geq 97\}$
- S = $\{x \neq y, x \neq z, y = c_{97}, z = c_{106}\}$

Example #2

$$\frac{\langle A_0, S_0 \rangle}{\mathbf{code(y)=97, code(z)=106 \in A}}\ \text{C-Intro (2)}$$

- A = {106≥code(x)≥97, code(y)=97, code(z)=106}
- S = {x≠y, x≠z, y=c_{97}, z=c_{106}}

Example #2

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\langle A_0, S_0\rangle}}{\texttt{code(y)=97, code(z)=106}\in S}\ \text{C-Intro (2)}}{\dots}}{\textbf{unsat}} \qquad \dfrac{\dfrac{\texttt{code(x)}\neq\texttt{code(y)}\in A}{\dots}}{\textbf{unsat}}\ \text{C-Inj} \qquad \dfrac{\dfrac{\dots}{\textbf{unsat}}}{\dots}\ \text{C-Inj}}{\textbf{code(x)}\neq\textbf{code(z)}\in\textbf{A}} \qquad \dfrac{\dots}{\textbf{unsat}}$$

- A = $\{106 \geq \texttt{code(x)} \geq 97, \texttt{code(y)}=97, \texttt{code(z)}=106, \texttt{code(x)} \neq \texttt{code(y)}, \texttt{code(x)} \neq \texttt{code(z)}\}$
- S = $\{x \neq y,\ x \neq z,\ y = c_{97},\ z = c_{106}\}$

Example #2

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\langle A_0, S_0\rangle}}{\texttt{code(y)=97, code(z)=106}\in S}\ \text{C-Intro (2)}}{\dfrac{\dots}{\textbf{unsat}} \quad \texttt{code(x)}\neq\texttt{code(y)}\in A \quad \dfrac{\dots}{\textbf{unsat}}}\ \text{C-Inj}}{\dfrac{\dots}{\textbf{unsat}} \quad \textbf{code(x)}\neq\textbf{code(z)}\in\textbf{A} \quad \dfrac{\dots}{\textbf{unsat}}}\ \text{C-Inj}}{}\ \text{C-Inj}$$

- Derivation tree with saturated configuration … ∴ $A_0 \wedge S_0$ is T_{AS}-satisfiable
  - Model for `x` determined by model for `code(x)`

Correctness Theorem

- Calculus is:
  - **Terminating** (all derivation trees are finite)
  - **Refutation-sound**
    - A closed derivation tree with root M $\Rightarrow$ M is T_{AS}-unsat
  - **Solution-sound**
    - A derivation tree with root M with a saturated configuration $\Rightarrow$ M is T_{AS}-sat
      - Proof is constructive, a model for M can be produced in all such cases

$\therefore$ Calculus is a decision procedure for T_{AS}

Implementation Details

- Implemented in CVC4 SMT solver
  - As part of its DPLL(T) solver for strings

- Combined with other procedures for strings
  - E.g. **[Liang et al CAV 2014, Liang et al FroCoS 2015, Reynolds et al CAV 2017]**
  - All rules of calculus with premises $S \models l$ become $S \models_T l$
    - Where $\models_T$ denotes entailment modulo theory T

- Strategy crafted in these combinations:
  - C-Valid, C-Collapse applied eagerly
  - C-Intro, C-Inj applied lazily

Reductions: Conversion Functions

- More efficient reductions that leverage `code`, including:

- Conversion between strings and integers `to_int(x)`:
  - ⊗ `… ite(x[i]="9",9,ite(x[i]="8",8, …ite(x[i]="0",0,-1)…)`
  - ⇒ `… ite(48≤code(x[i])≤57,code(x[i])-48, -1)`

- Conversion between lowercase and uppercase strings `to_lower(x)`:
  - ⊗ `… ite(x[i]="A","a",ite(x[i]="B","b", … ite(x[i]="Z","z",x[i])…)`
  - ⇒ `… code(x[i])+ite(65≤code(x[i])≤90,32,0)`

Reductions: Ordering, RegExp Range

- Lexicographic ordering:
  - ⊗ $x \le y \Leftrightarrow \exists i \ldots (x[i]=y[i] \vee (x[i]=\text{“A”} \wedge y[i]=\text{“B”}) \vee (x[i]=\text{“A”} \wedge y[i]=\text{“C”}) \ldots)$
  - ⇒ $x \le y \Leftrightarrow \exists i \ldots \mathbf{code}(x[i]) \le \mathbf{code}(y[i])$

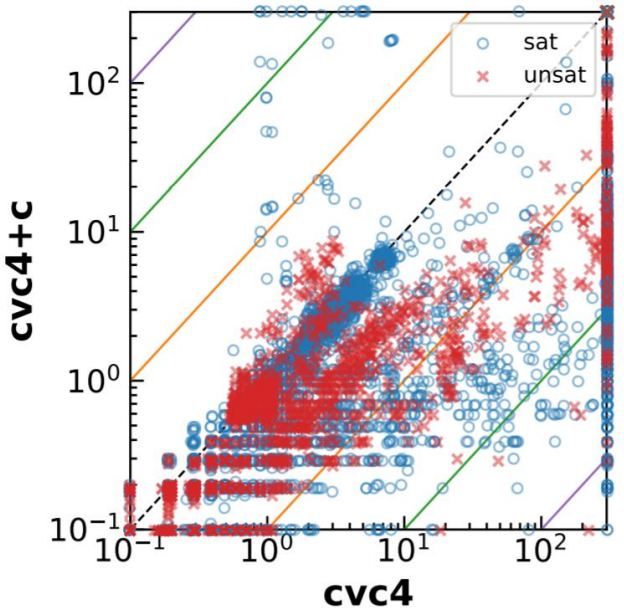
- Regular expression ranges:
  - ⊗ $x \in \text{range}(c_1, c_2) \Leftrightarrow \text{len}(x)=1 \wedge (x=c_1 \vee \ldots \vee x=c_2)$
  - ⇒ $x \in \text{range}(c_1, c_2) \Leftrightarrow \mathbf{code}(c_1) \le \mathbf{code}(x) \le \mathbf{code}(c_2)$

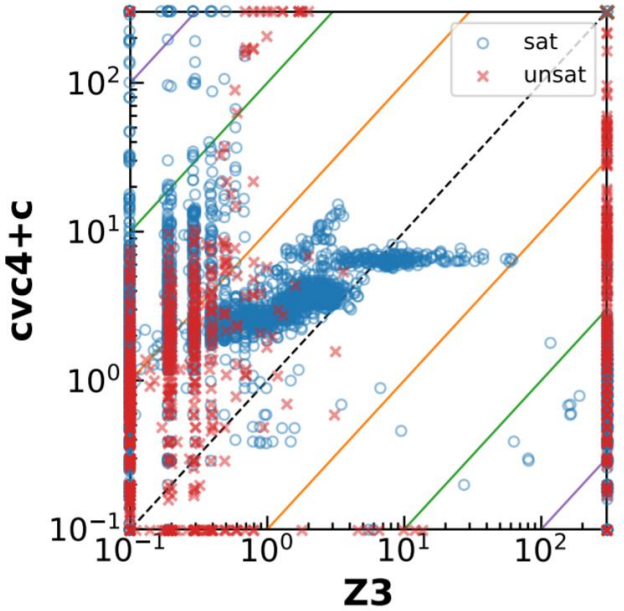
Experiments

- Compared solvers
  - **cvc4+c** : new version, includes techniques from this paper
  - **cvc4:** previous version
  - **z3**
- 21573 benchmarks from PyConByte
  - Correspond to concolic execution of Python code
  - Contain applications of `to_int`, `from_int`, regular expression ranges
- 300 second timeout

Experimental Results

Benchmark Set		cvc4+c	cvc4	z3
py-conbyte_cvc4	sat	**1344**	1104	1187
	unsat	**8576**	8547	8482
	×	13	282	264
py-conbyte_trauc	sat	**1009**	929	697
	unsat	1424	1407	**1428**
	×	13	110	321
py-conbyte_z3seq	sat	**1354**	1126	1343
	unsat	**5864**	5797	5719
	×	35	330	191
py-conbyte_z3str	sat	**711**	652	692
	unsat	**1227**	1223	1223
	×	3	66	26
Total	sat	**4418**	3811	3919
	unsat	**17091**	16974	16852
	×	64	788	802





- 10x t/o reduction
- Faster runtimes
- Improvement wrt state-of-the-art

Summary

- New decision procedure for string-to-code point conversion
- Efficient reduction techniques
  - Conversion functions, lexicographic ordering, regular expression ranges
- Implemented in CVC4
  - Combined with other procedures
- Experiments: significant improvement wrt SMT solvers for strings

Future Work

- Support for more extended functions of interest to user applications
- Proof production for strings