Challenges for Fast Synthesis Procedures in SMT

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Synthesis

- SMT solvers act as *subroutines* for automated synthesis
  - For program snippets, planning, digital circuits, programming by examples, …
- More recently, SMT solvers act as *stand-alone tools* for synthesis
  - Leveraging their support for first-order quantification

[Reynolds et al CAV2015]

Synthesis Conjectures

$$\exists f.\forall x.P(f,x)$$

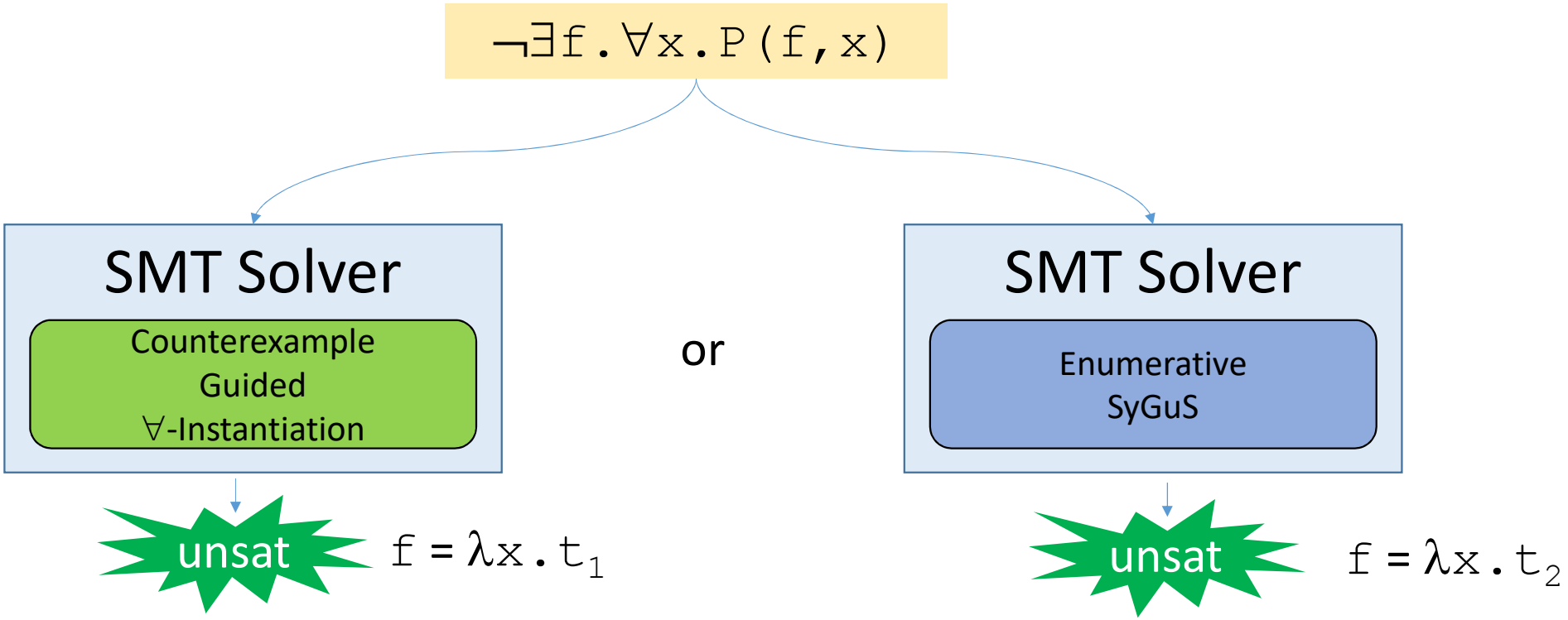
There exists a function `f` for which property `P` holds for all `x`

Refutation-Based Synthesis in SMT

$\neg\exists f.\forall x.P(f,x)$

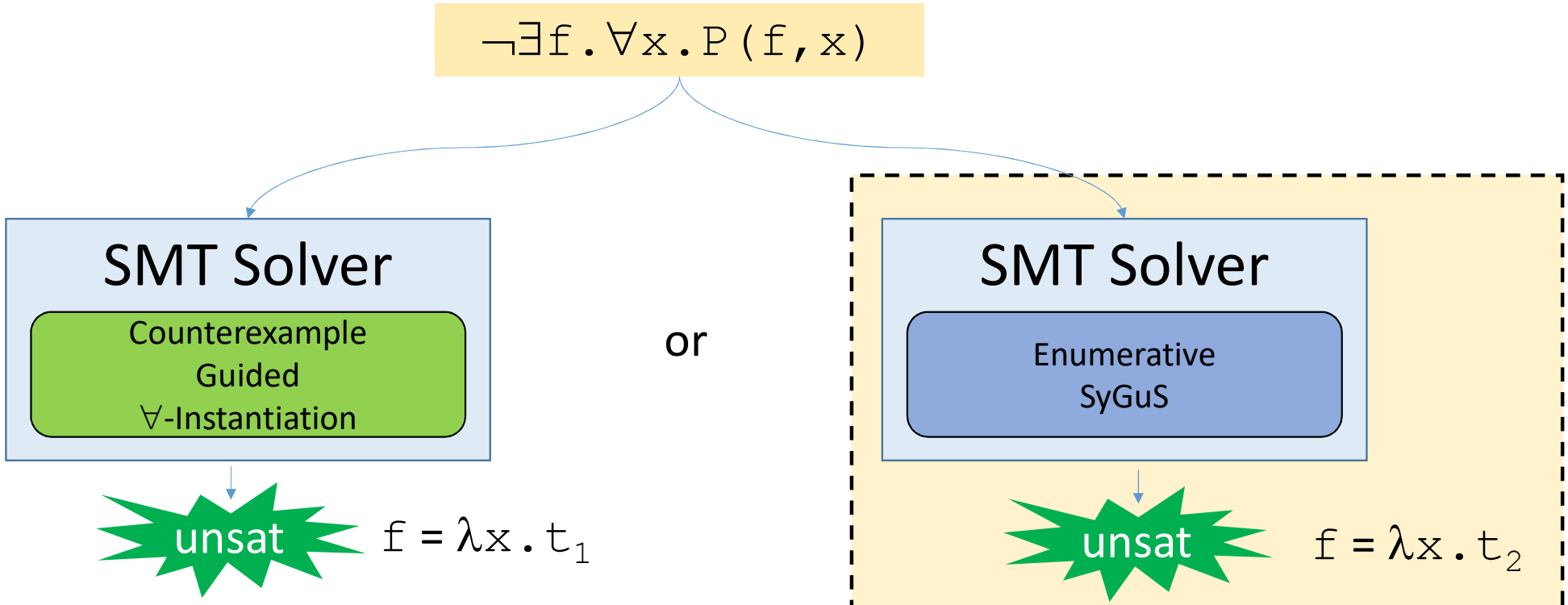
(**negated** synthesis conjecture)

Refutation-Based Synthesis in SMT



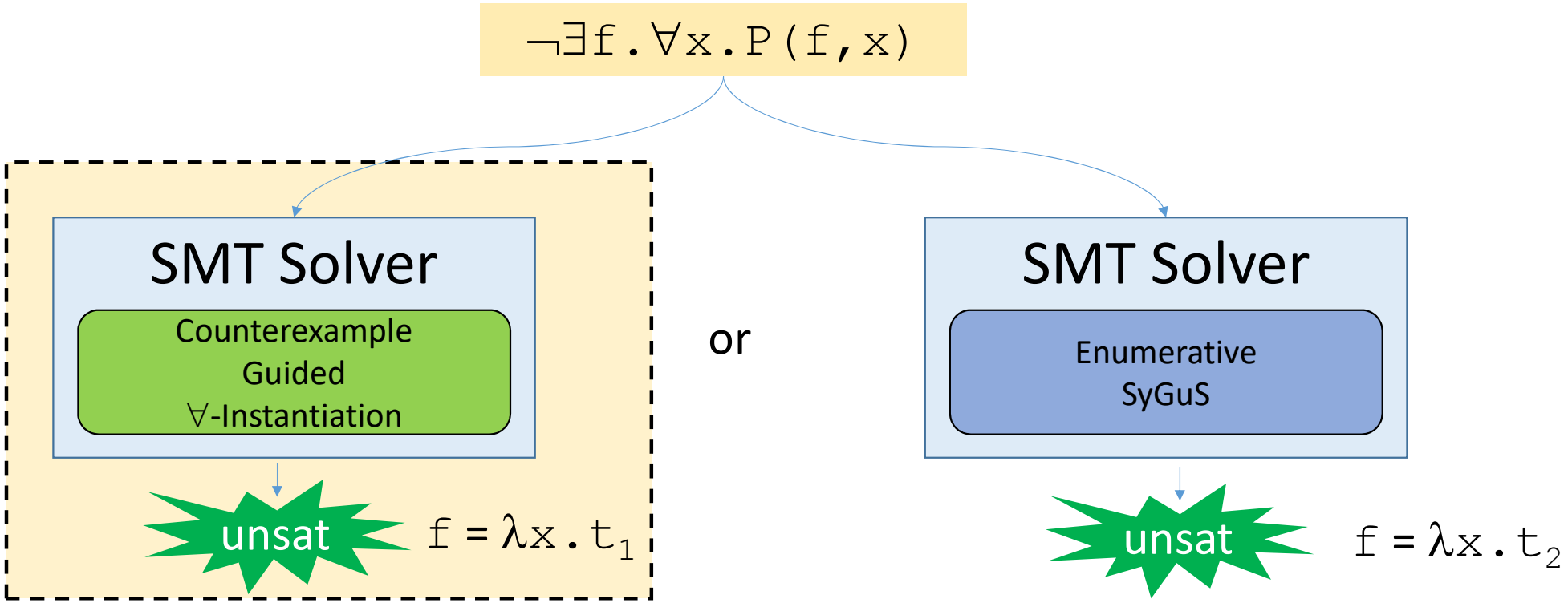
- Two approaches for refutation-based synthesis in SMT solvers **[Reynolds et al CAV2015]**

Refutation-Based Synthesis in SMT



⇒ Based on enumerative search (via syntax-guided synthesis) **[Alur et al 2013]**

Refutation-Based Synthesis in SMT



⇒ Based on first-order quantifier instantiation **(focus of this talk)**

Single Invocation Conjectures

- Some synthesis conjectures are *essentially first-order*:

$$\neg\exists f.\forall xy.\ \mathbf{f(x,y)} \geq x \wedge \mathbf{f(x,y)} \geq y \wedge (\mathbf{f(x,y)} = x \vee \mathbf{f(x,y)} = y)$$

"**f(x,y)** is the maximum of x and y"

Single Invocation Conjectures

$\neg\exists f.\forall xy.\ \mathbf{f(x,y)} \geq x \wedge \mathbf{f(x,y)} \geq y \wedge (\mathbf{f(x,y)} = x \vee \mathbf{f(x,y)} = y)$

$\mathtt{Int} \times \mathtt{Int} \rightarrow \mathtt{Int}$

All occurrence of f are in terms of the form **f(x,y)**

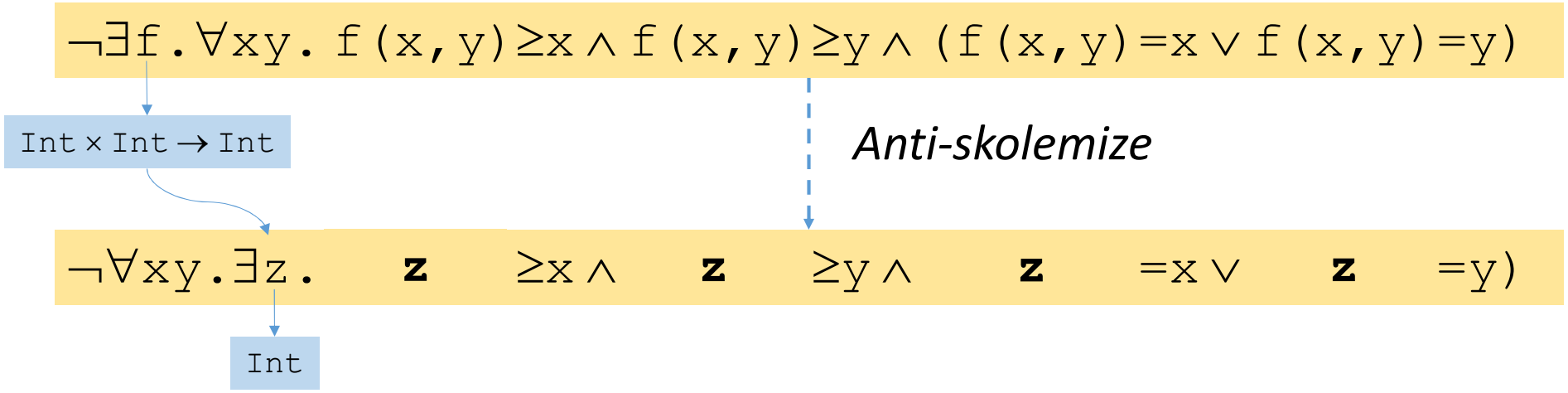
$\Rightarrow$ "single invocation" synthesis conjectures

Single Invocation Conjectures

$$\neg\exists f.\forall xy.\ f(x,y)\geq x \wedge f(x,y)\geq y \wedge (f(x,y)=x \vee f(x,y)=y)$$

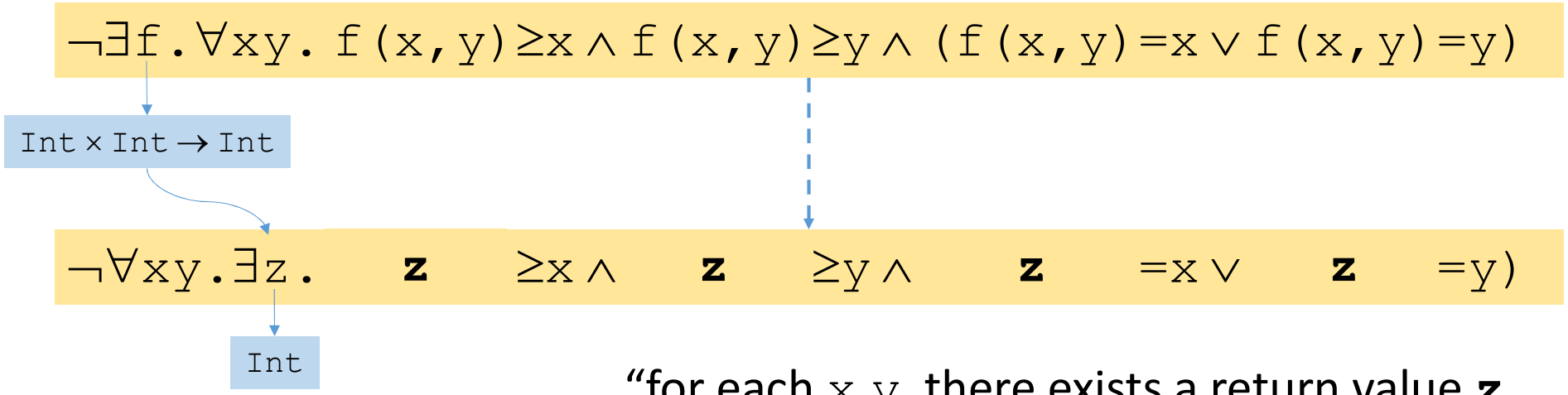
$$\texttt{Int} \times \texttt{Int} \rightarrow \texttt{Int}$$

Single Invocation Conjectures



[Reynolds et al CAV2015]

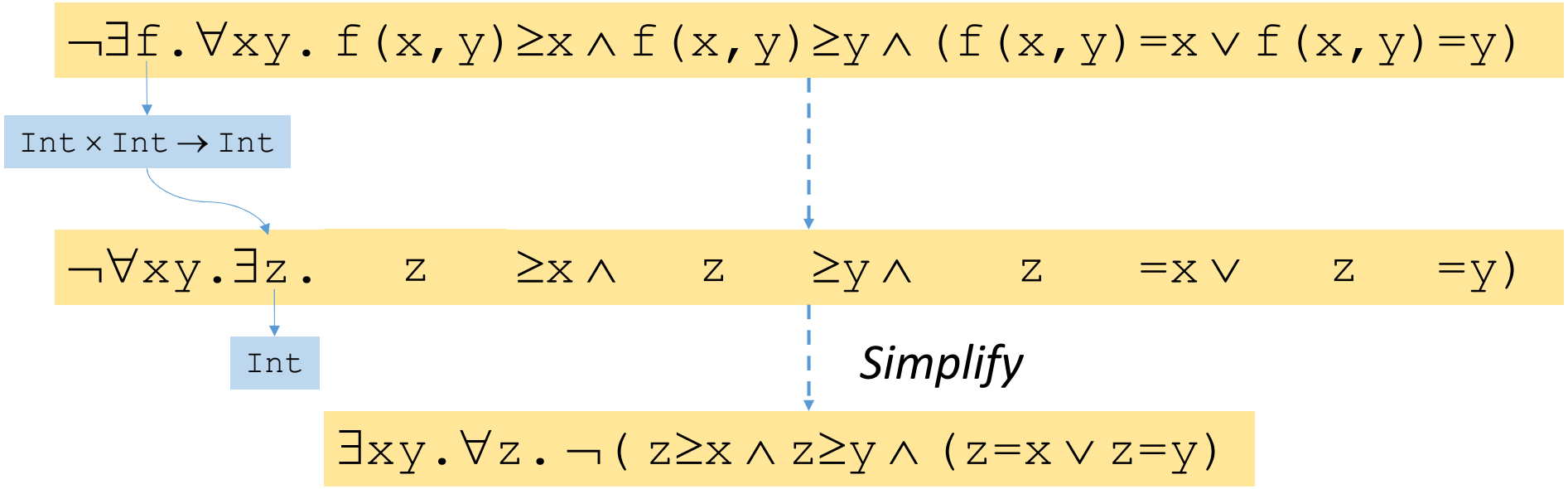
Single Invocation Conjectures



"for each x,y, there exists a return value **z** that is the maximum of x and y"

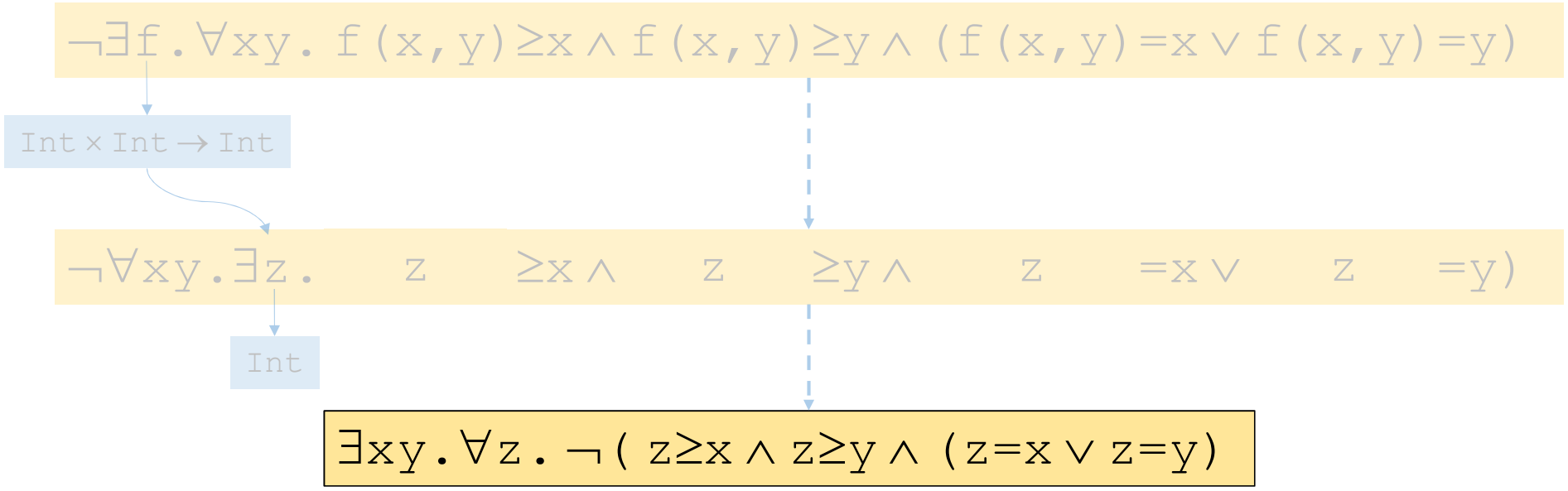
[Reynolds et al CAV2015]

Single Invocation Conjectures



[Reynolds et al CAV2015]

Single Invocation Conjectures



First-order linear arithmetic $\Rightarrow$ *Solvable by first-order* $\forall$*-instantiation*

[Reynolds et al CAV2015]

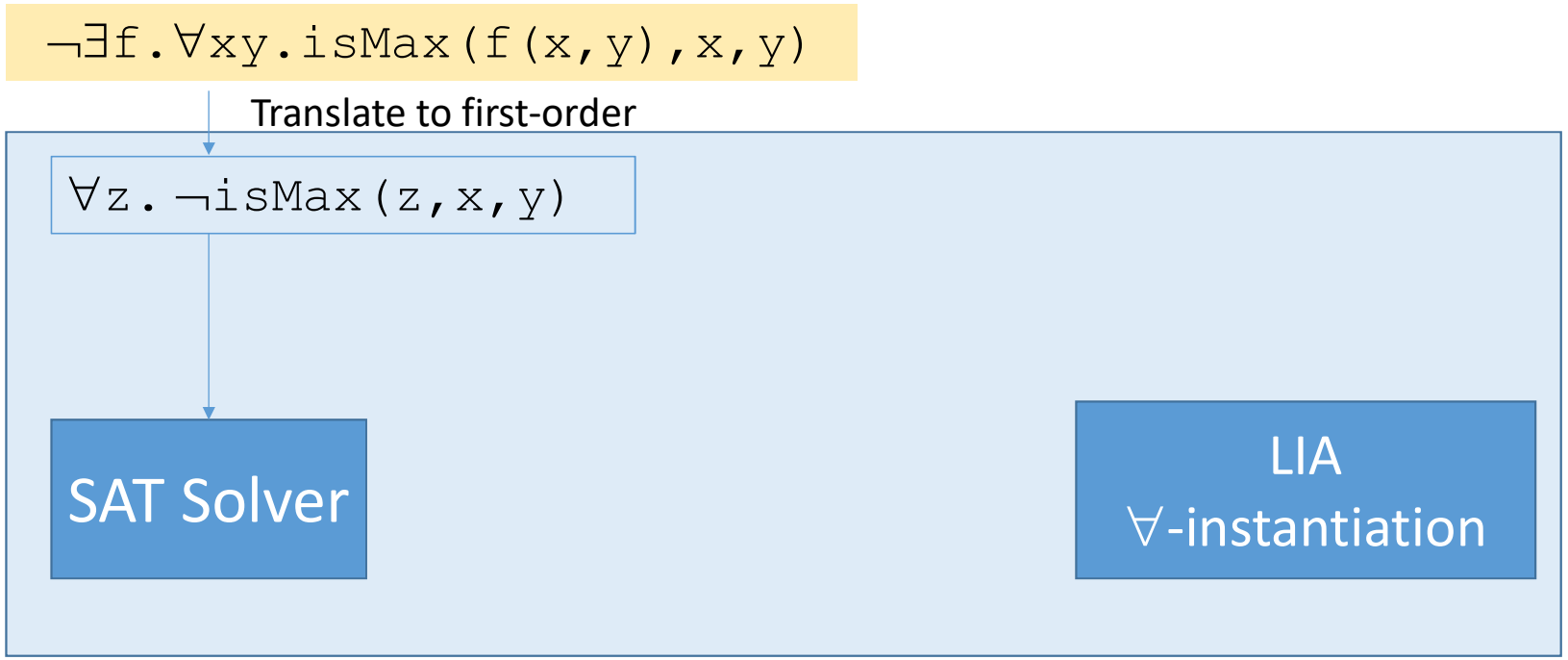
Single Invocation Synthesis in SMT

$\neg\exists f.\forall xy.\ f(x,y)\geq x \wedge f(x,y)\geq y \wedge (f(x,y)=x \vee f(x,y)=y)$

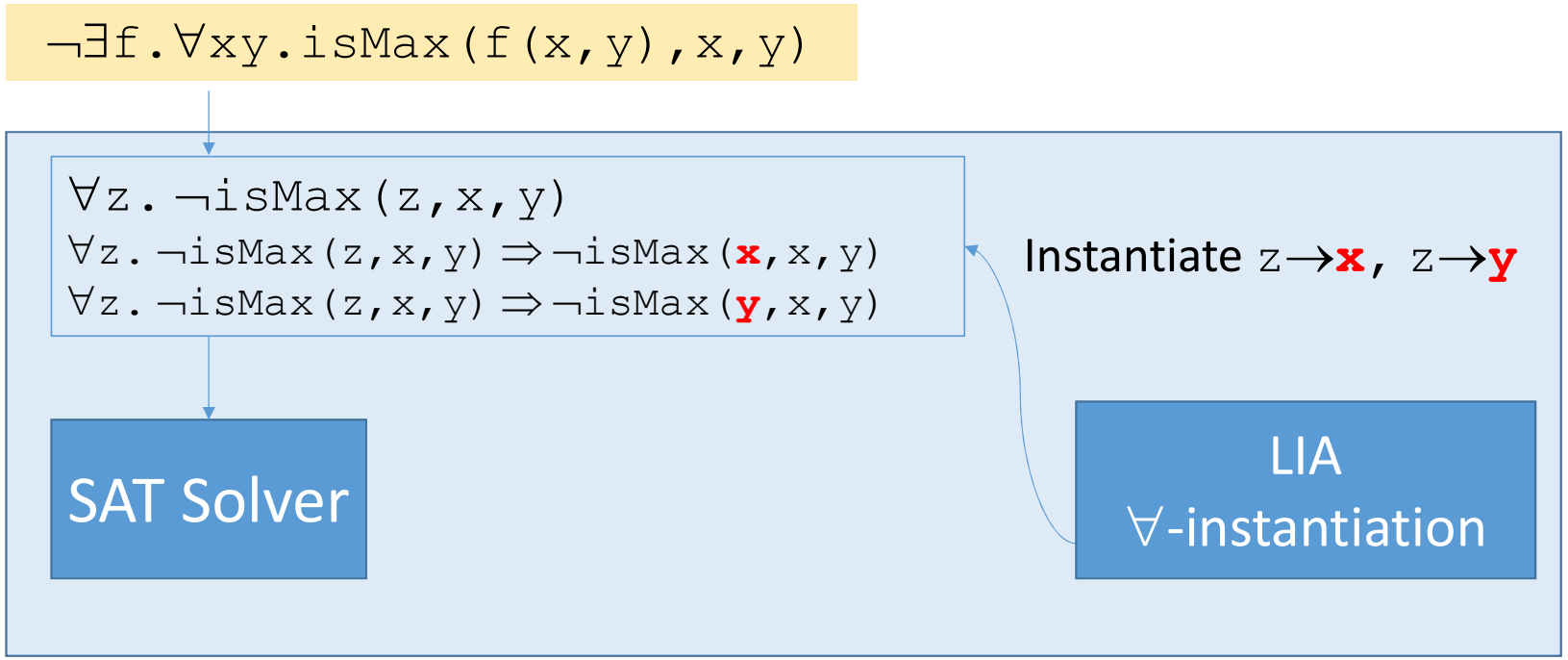
Single Invocation Synthesis in SMT

$\neg\exists f.\forall xy.\mathtt{isMax}(f(x,y),x,y)$

Single Invocation Synthesis in SMT



Single Invocation Synthesis in SMT



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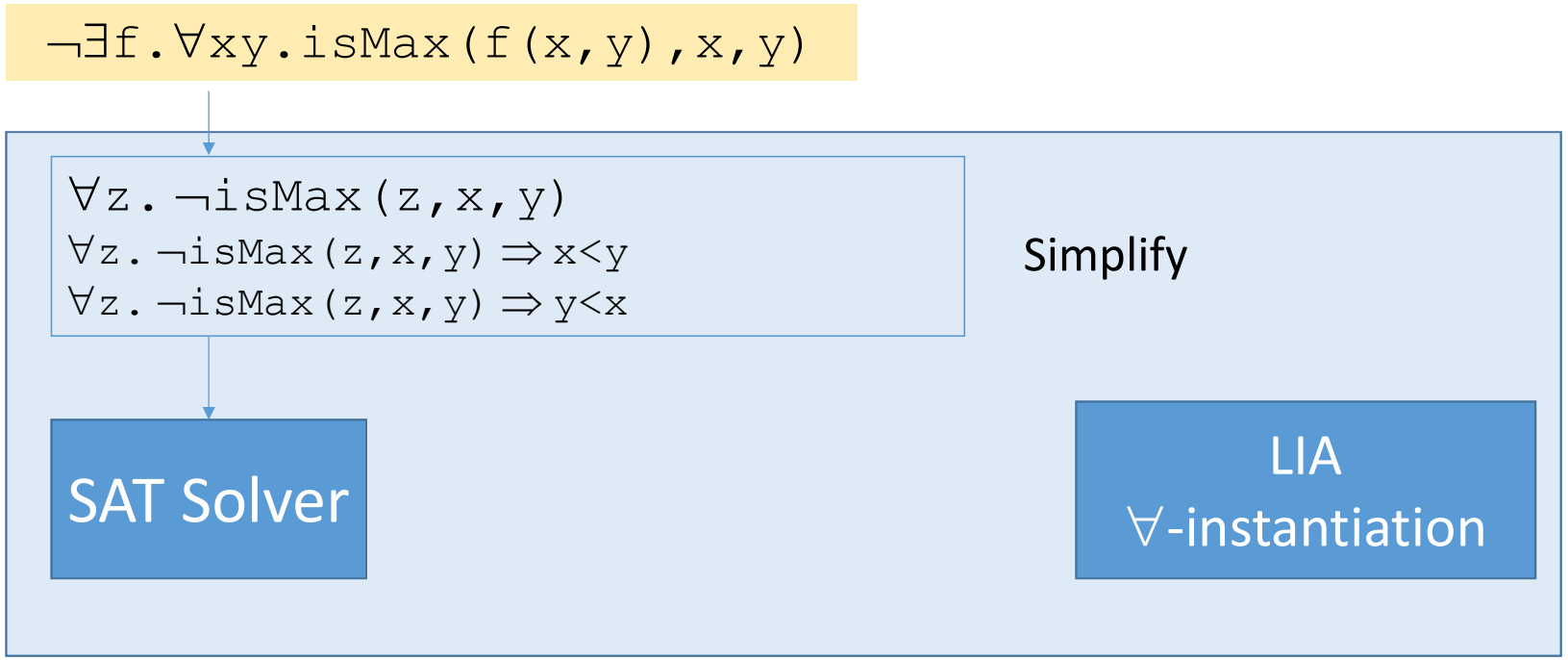

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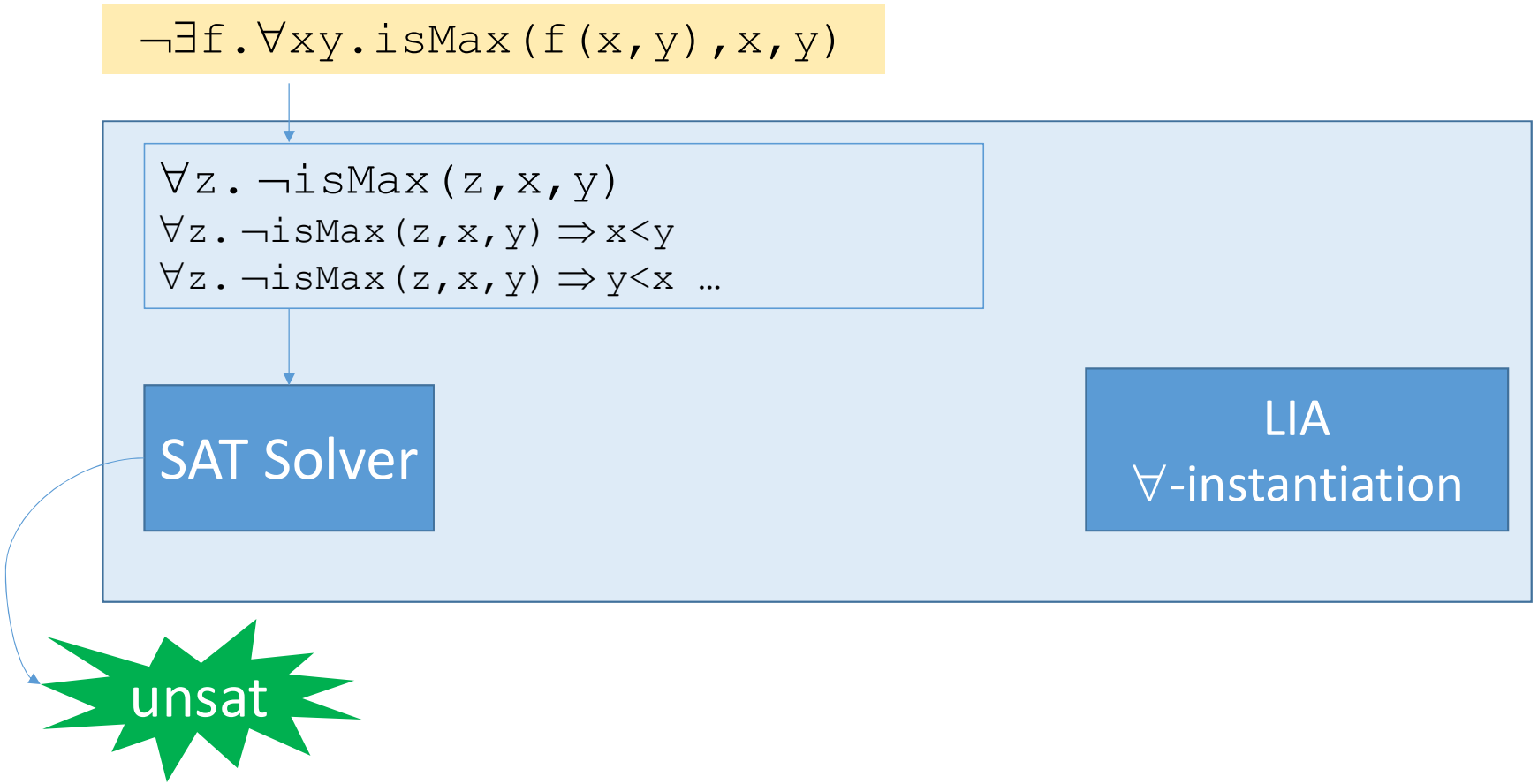

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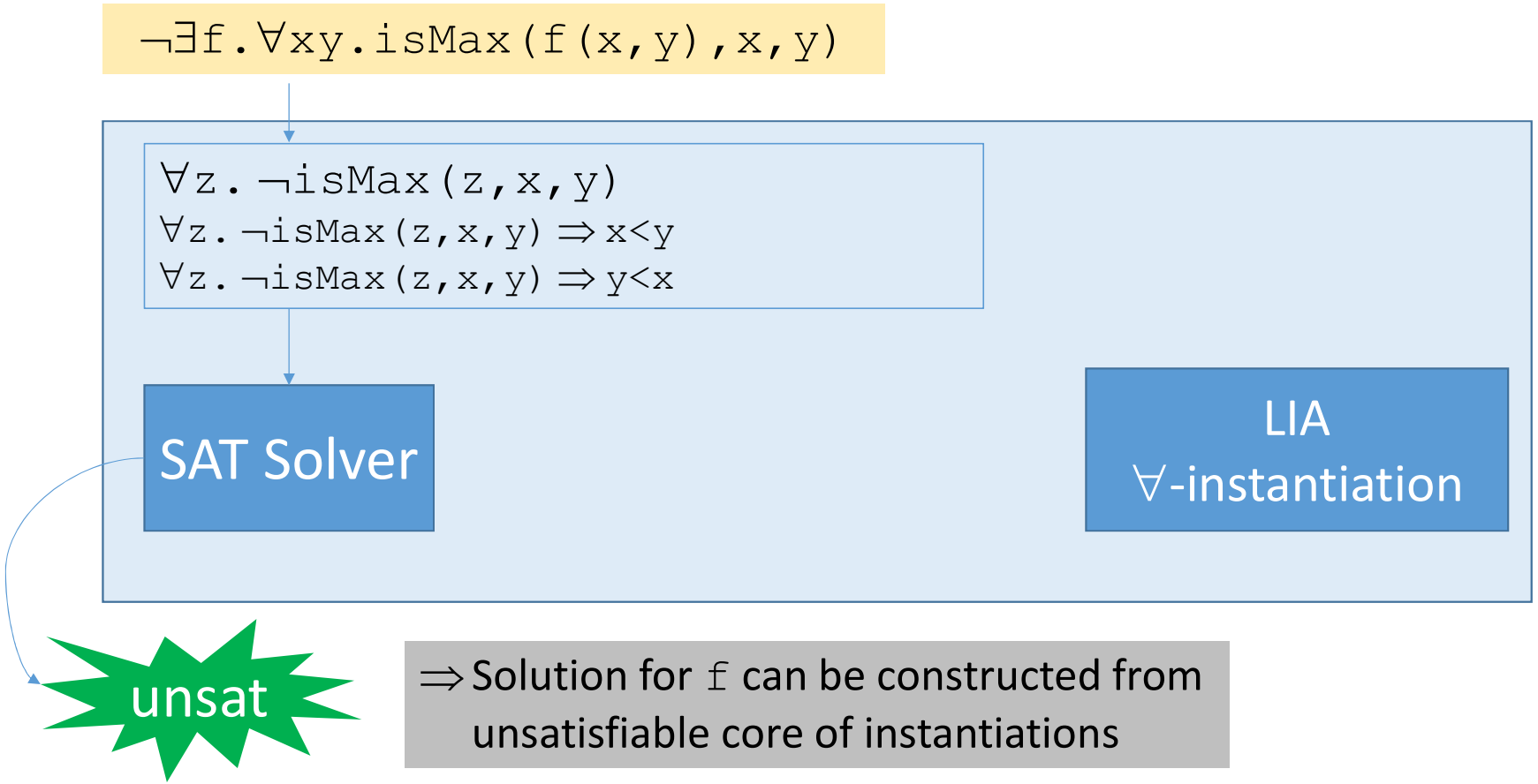
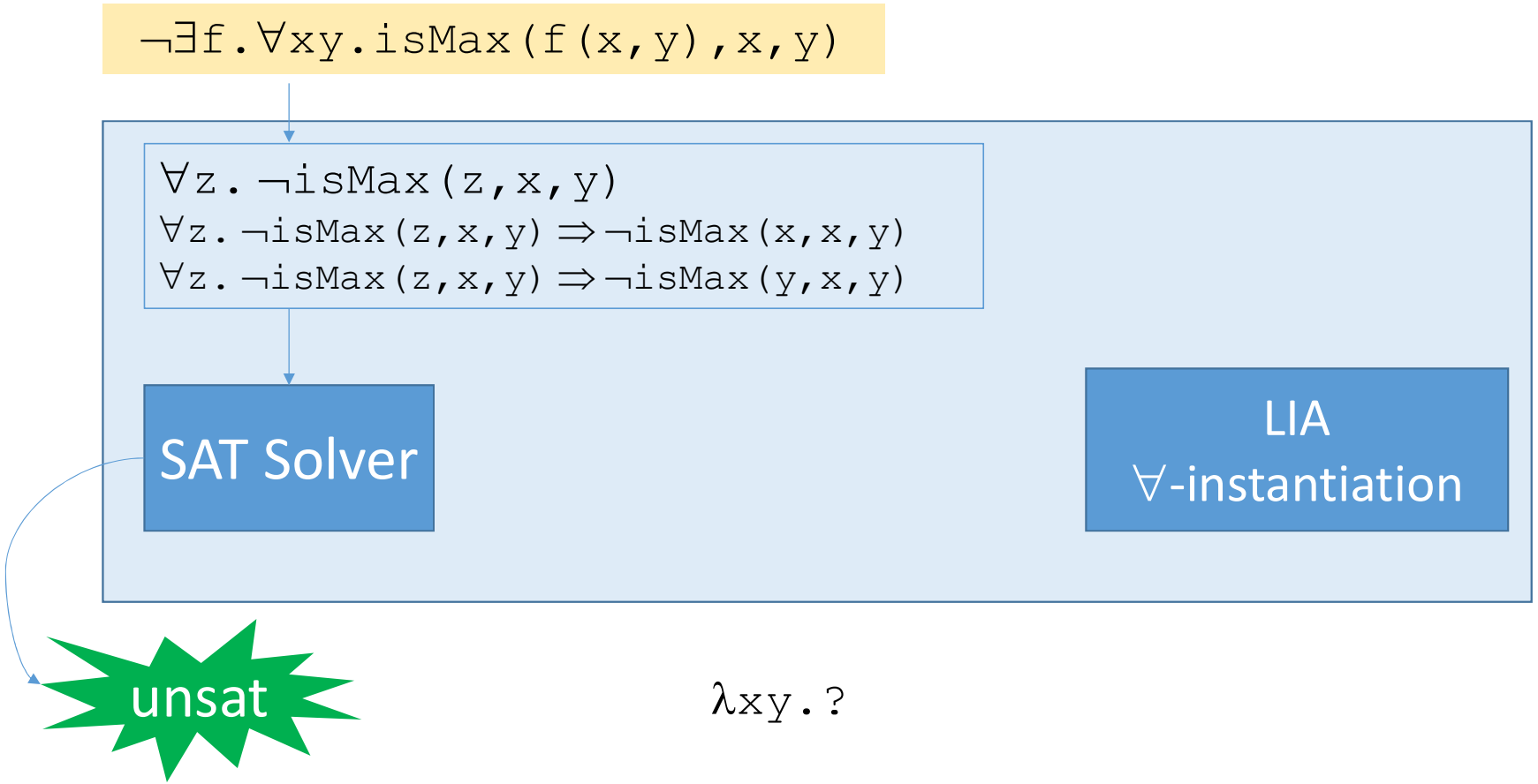


⇒ Solution for f can be constructed from unsatisfiable core of instantiations

Single Invocation Synthesis in SMT



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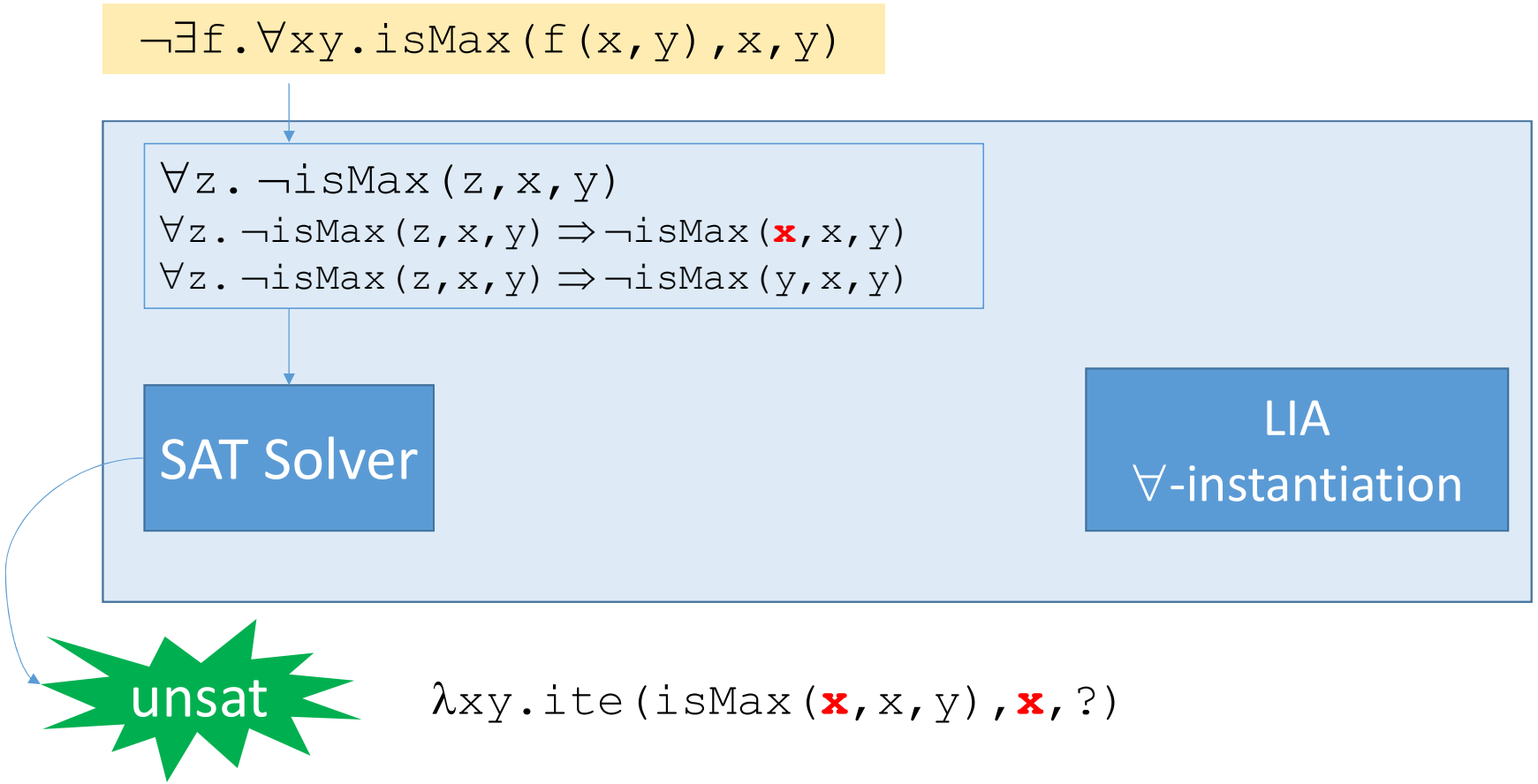

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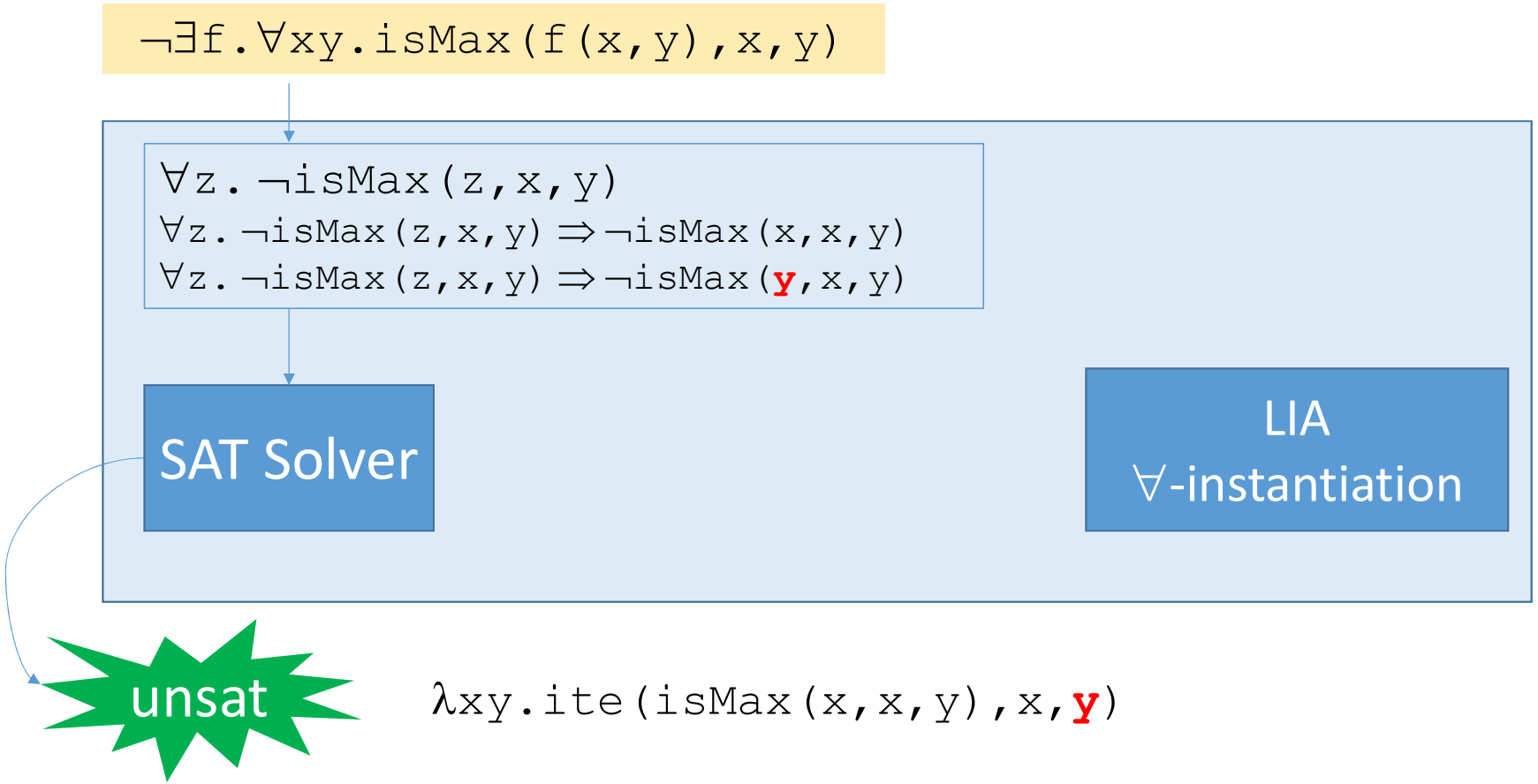

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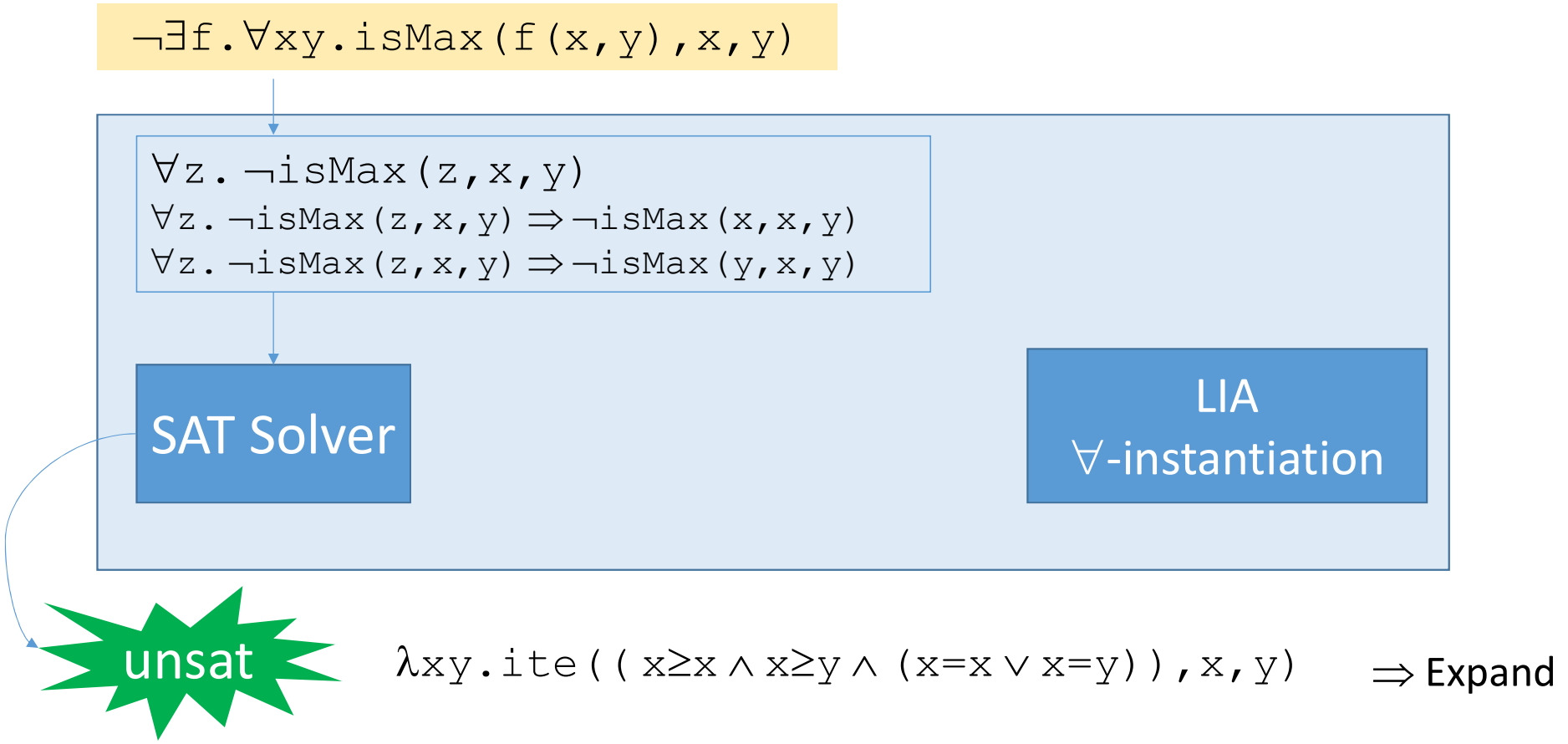

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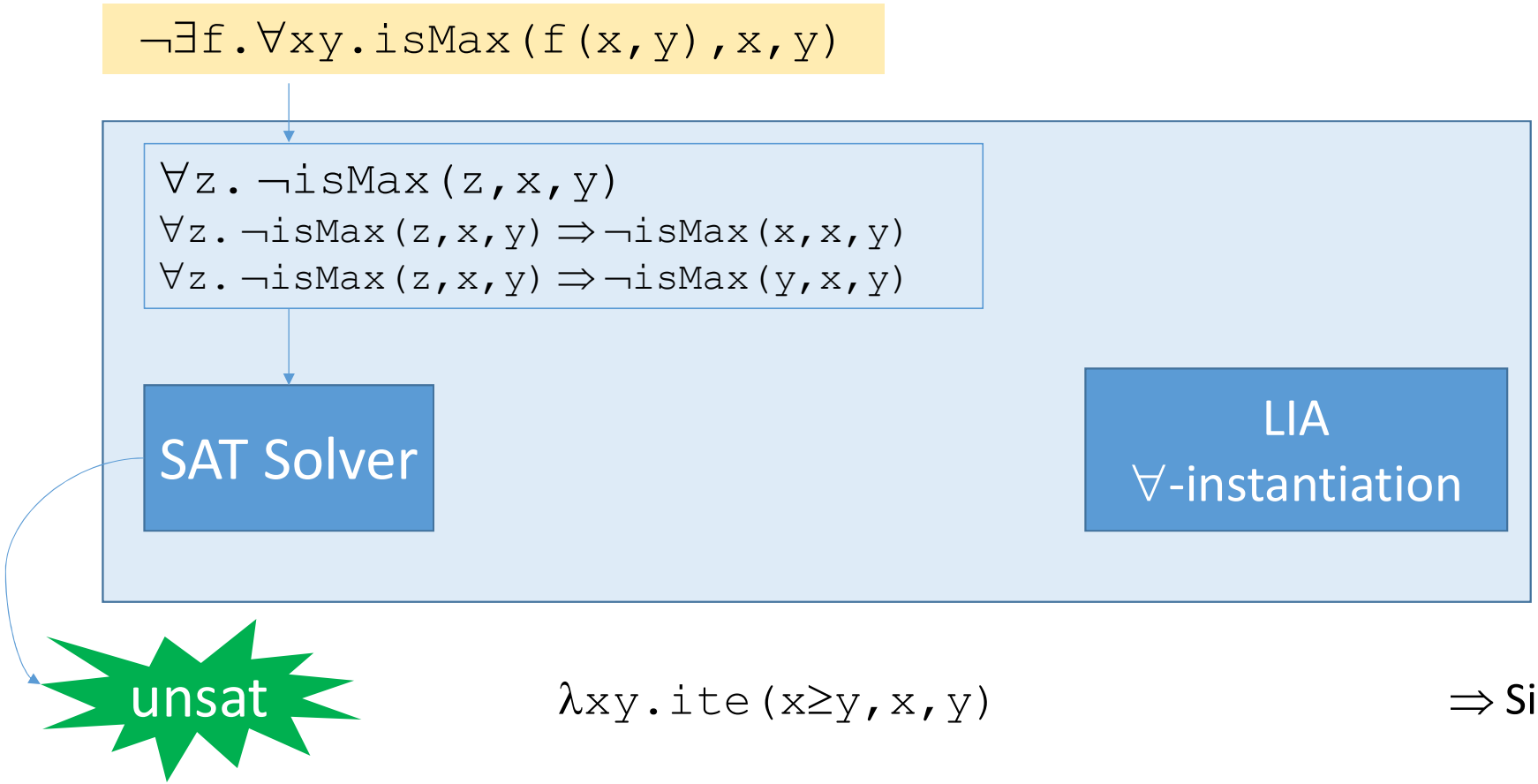

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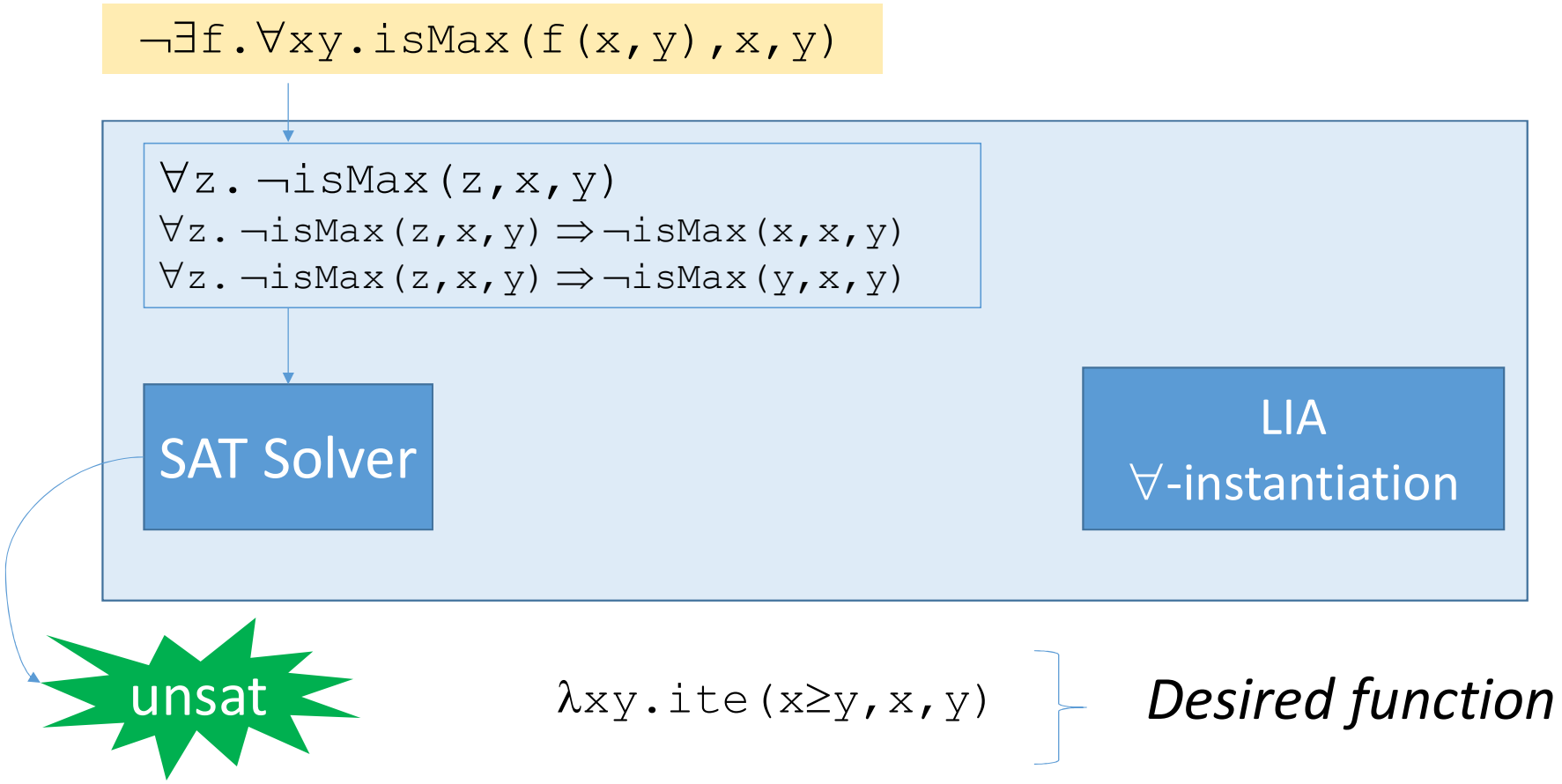

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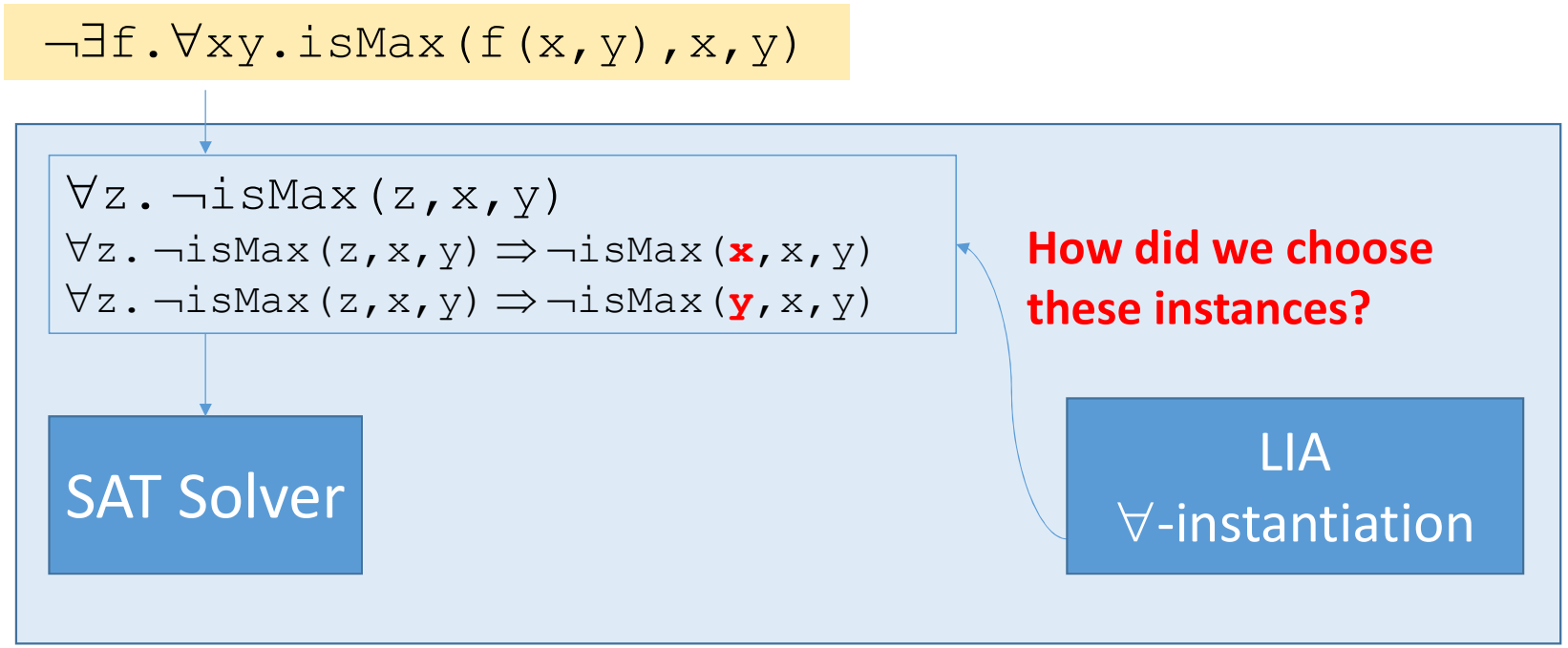

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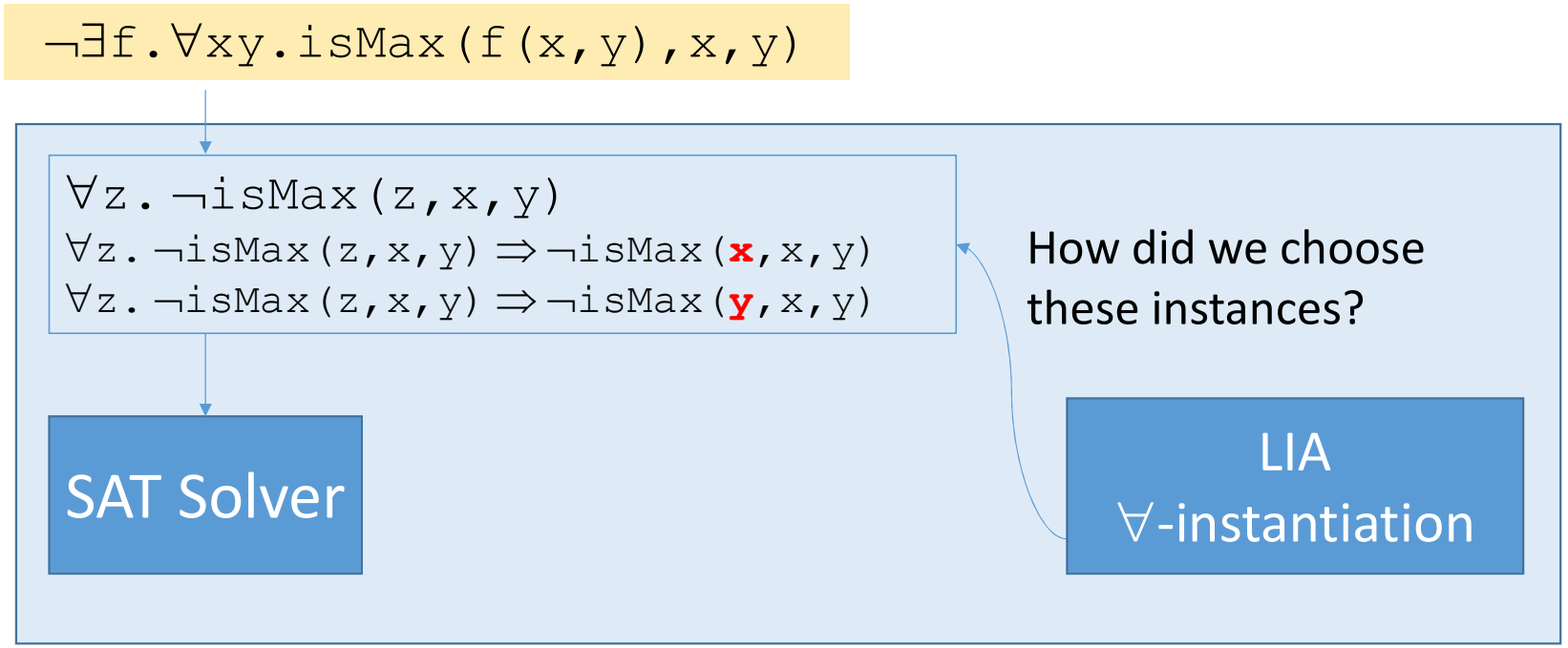

⇒ Use *counterexample-guided quantifier instantiation* (CEGQI)

Variants used in **[Monniaux 2010, Komuravelli et al 2014, Reynolds et al 2015, Dutertre 2015, Bjorner/Janota 2016, Fedyukovich et al 2016, Preiner et al 2017]**

Counterexample-Guided ∀-Instantiation

Quantifier Elimination Procedures

$\Leftarrow(\Rightarrow)^{?}$

Instantiation-Based procedures for $\exists\forall$ formulas

$\Leftarrow\Rightarrow$

Synthesis procedures for single-invocation properties

Counterexample-Guided ∀-Instantiation

- SMT+∀ linear arithmetic **[Monniaux 2010, Reynolds et al 2015, Dutertre 2015, Bjorner/Janota 2016]**
  - Based on maximal lower (minimal upper) bounds
    Analogous to **[Loos+Wiespfenning 93]**
  - Based on interior point method:
    Analogous to **[Ferrante+Rackoff 79]**
  - For integers: based on maximal lower (minimal upper) bounds (+⊂)
    Analogous to **[Cooper 72]**
- SMT + ∀ BV, QBF, finite domains **[Wintersteiger et al 2013, Rabe et al 2016, Preiner et al 2017]**
  - Based on model value, SyGuS, others?
- SMT + ∀ Strings, sets, floating points, datatypes
  - ???

***Finite** instantiation strategy ⇔ **sound** and **complete** synthesis procedure for s.i.*

Counterexample-Guided ∀-Instantiation

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  - ???

CHALLENGE #1:
How do we develop instantiation procedures for new SMT *theories*?

Finite *instantiation strategy* ⇔ ***sound*** *and* ***complete*** *synthesis procedure for s.i.*

Comparison of Synthesis Approaches

- *SMT + ∀-instantiation*
  - **Pro:** Very fast
  - **Pro:** Complete for (in)feasibility
  - **Con:** Non-optimal solutions
  - **Con:** Only for single-invocation conjectures
- Enumerative Search
  - **Con:** Typically very slow
  - **Con:** Cannot show infeasibility
  - **Pro:** Optimal (shortest) solutions
  - **Pro:** Applies to all second-order conjectures

Comparison of Synthesis Approaches

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CHALLENGES

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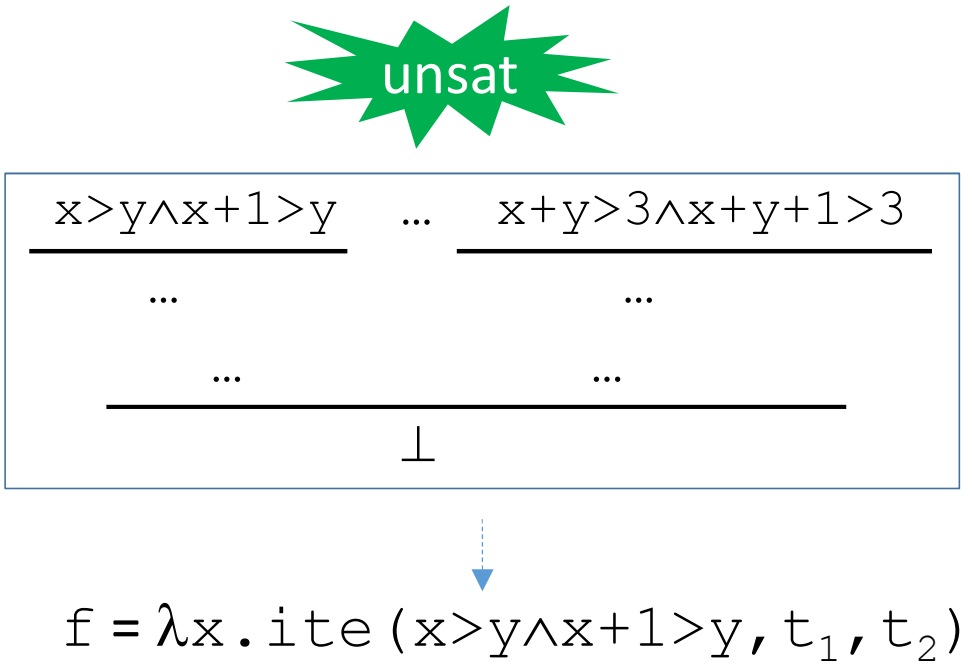
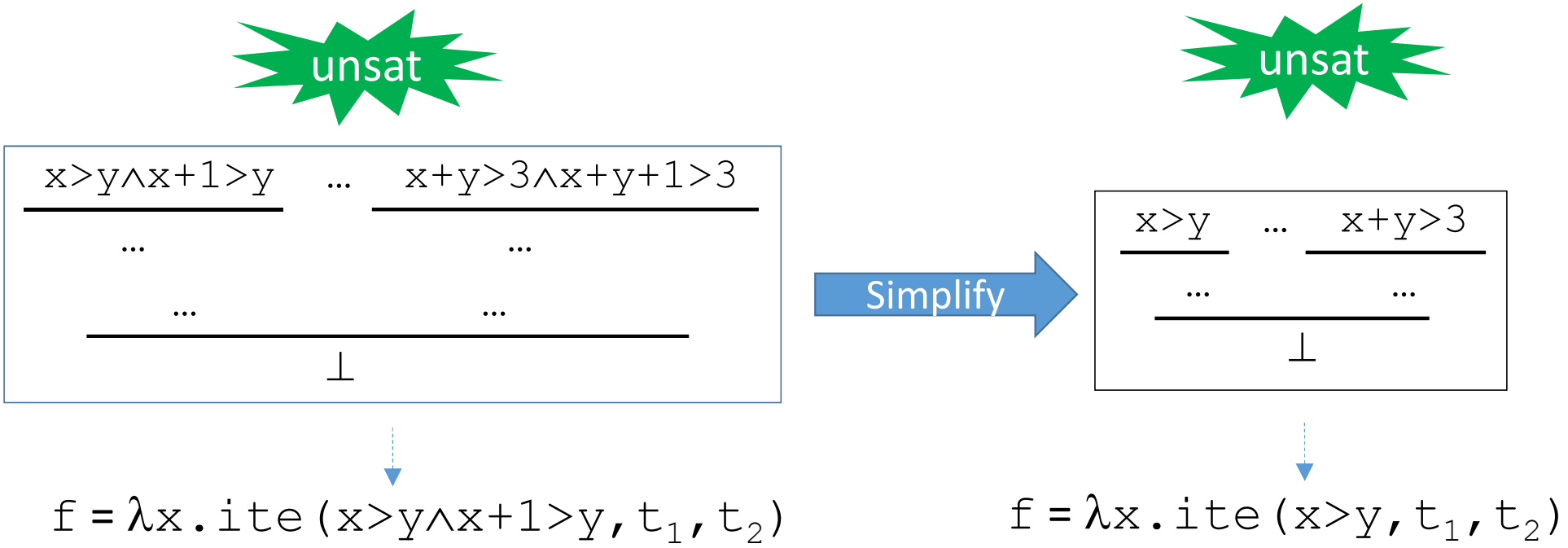
Shorter Solutions via Proof Analysis



Shorter Solutions via Proof Analysis



CHALLENGE #2:

What *proofs/analysis* techniques are relevant for solution minimization?

What if conjecture is *Partially Single Invocation?*

$$\exists I.\forall xx'.(pre(x)\Rightarrow I(x))\wedge((I(x)\wedge T(x,x'))\Rightarrow I(x'))\wedge(I(x)\Rightarrow post(x))$$

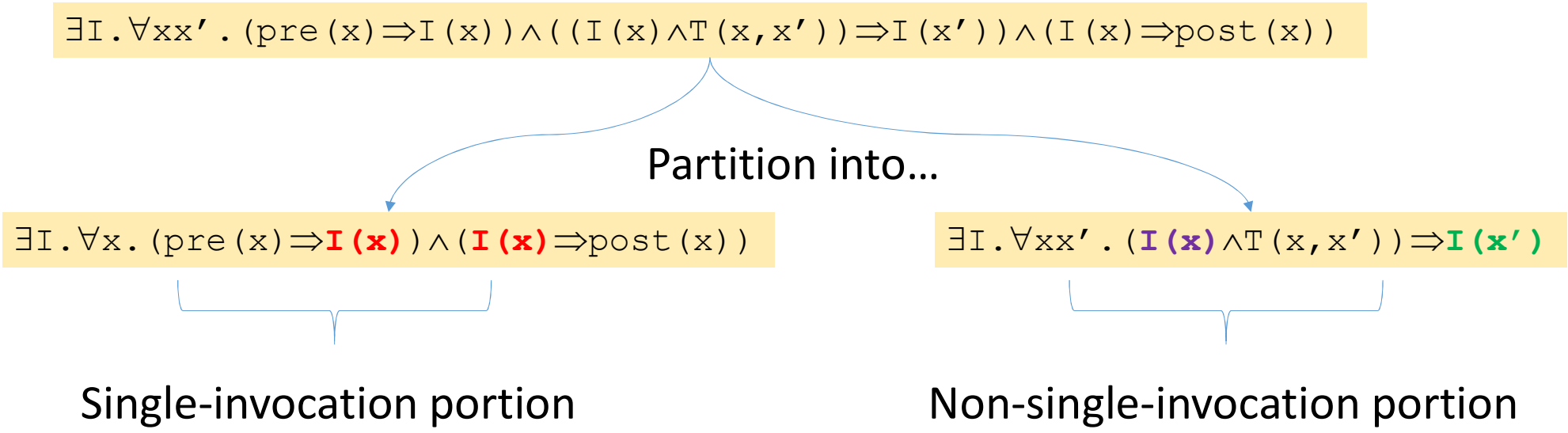
- Can single invocation techniques be leveraged beyond *single-invocation* conjectures?

What if conjecture is *Partially Single Invocation?*

$$\exists I.\forall xx'.(pre(x)\Rightarrow I(x))\wedge((I(x)\wedge T(x,x'))\Rightarrow I(x'))\wedge(I(x)\Rightarrow post(x))$$

E.g. invariant synthesis problem for `I` w.r.t `pre`, `T`, `post`

What if conjecture is *Partially Single Invocation?*



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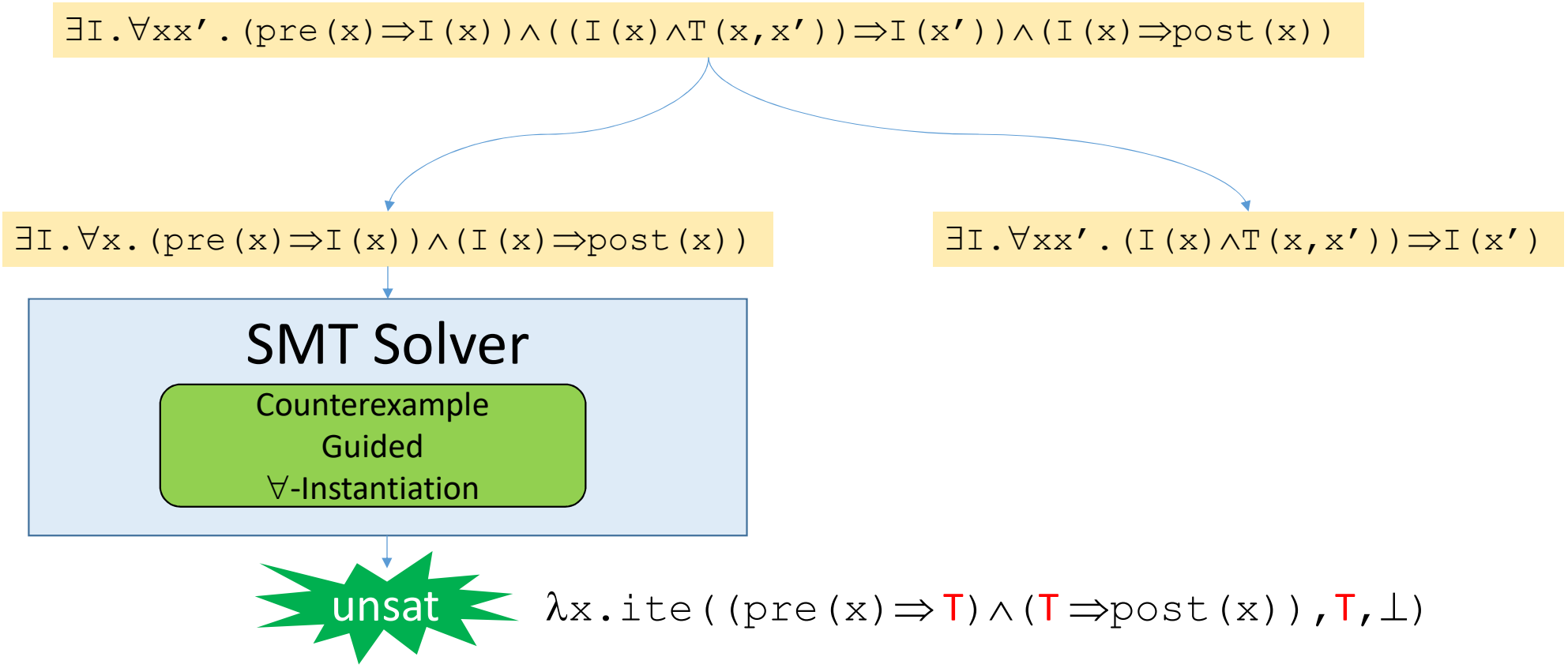

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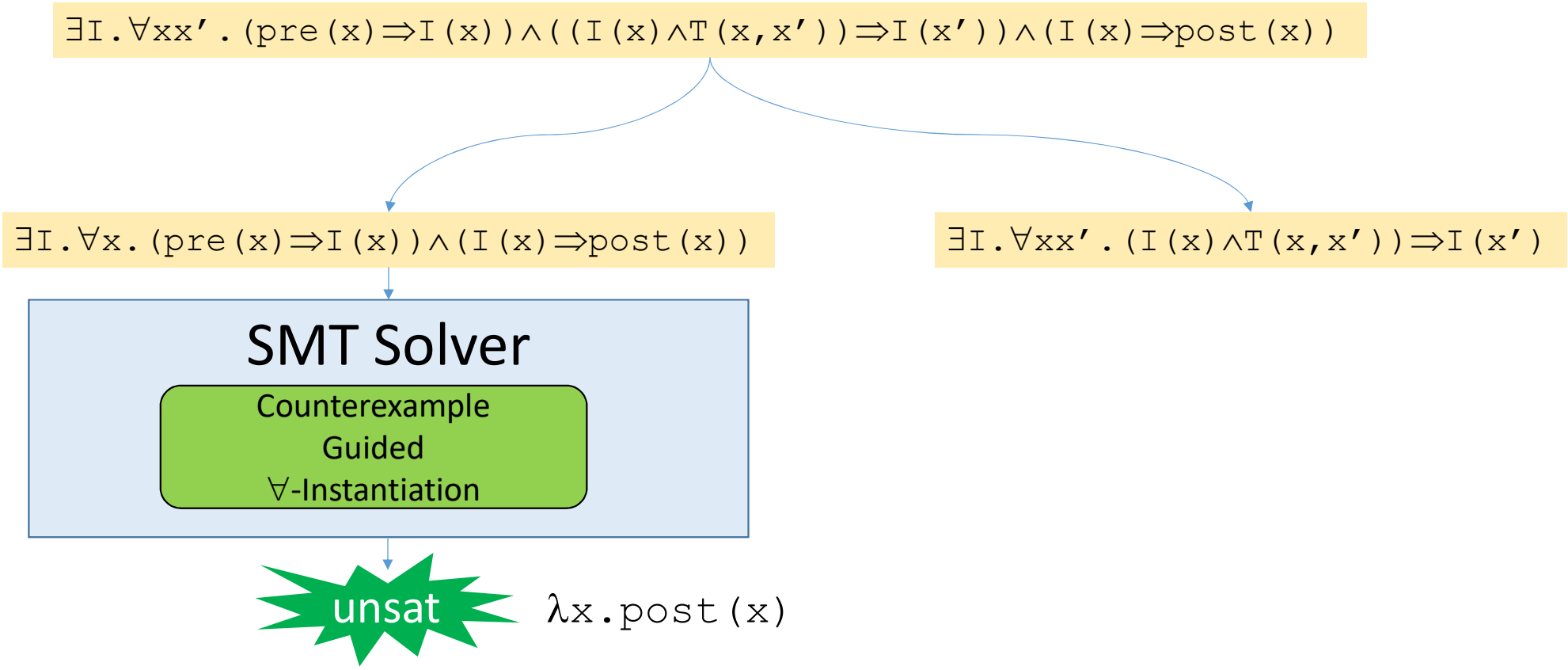

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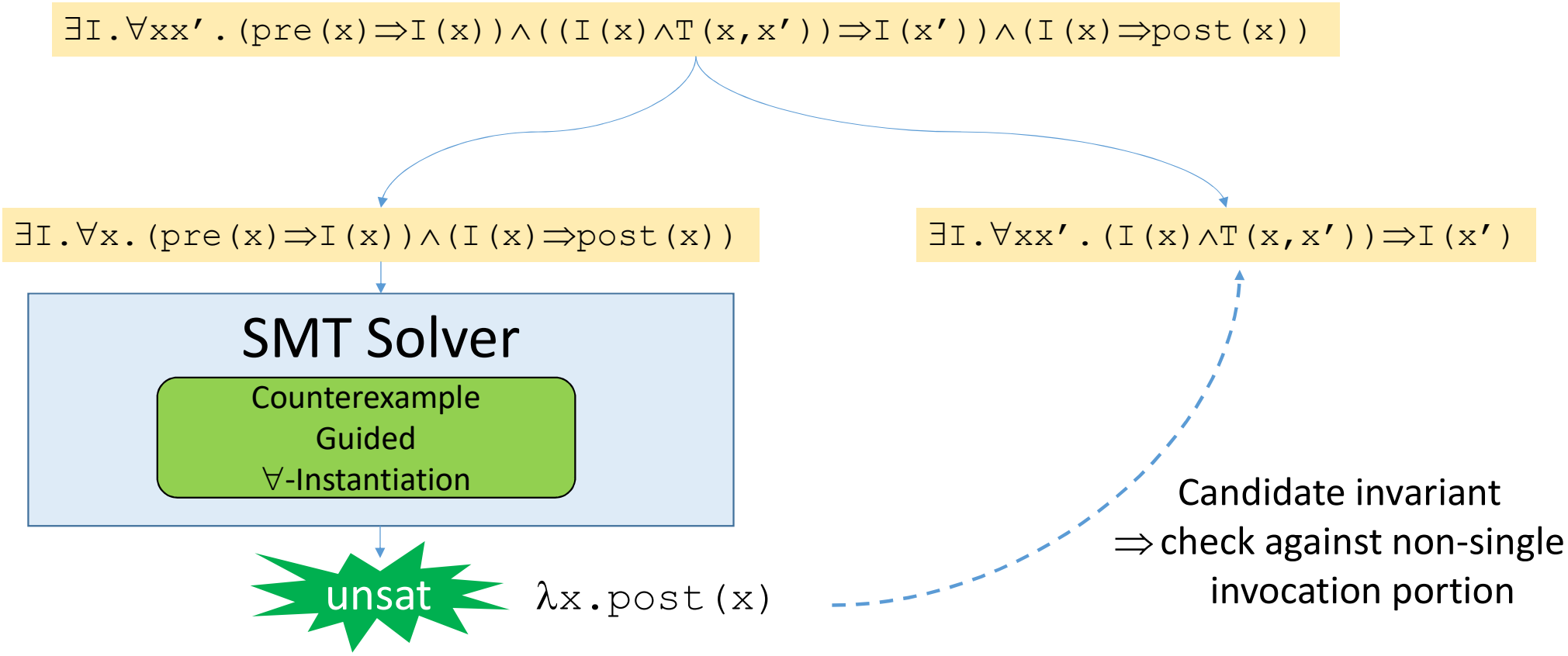

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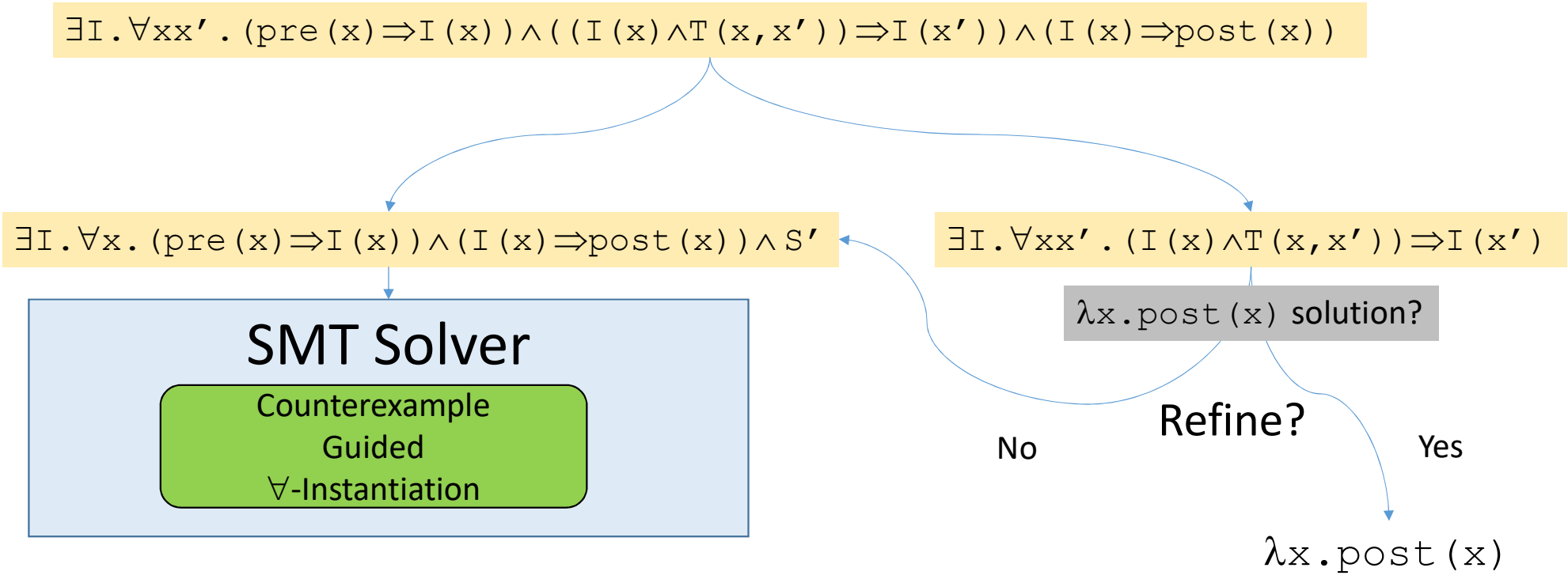


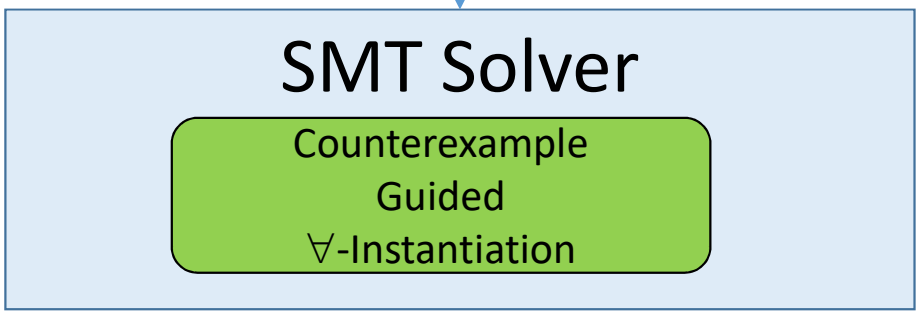
⇒Related to property-directed reachability (PDR) **[Bradley 2011]**

What if conjecture is *Partially Single Invocation?*

$\exists I.\forall xx'.(pre(x)\Rightarrow I(x))\wedge((I(x)\wedge T(x,x'))\Rightarrow I(x'))\wedge(I(x)\Rightarrow post(x))$

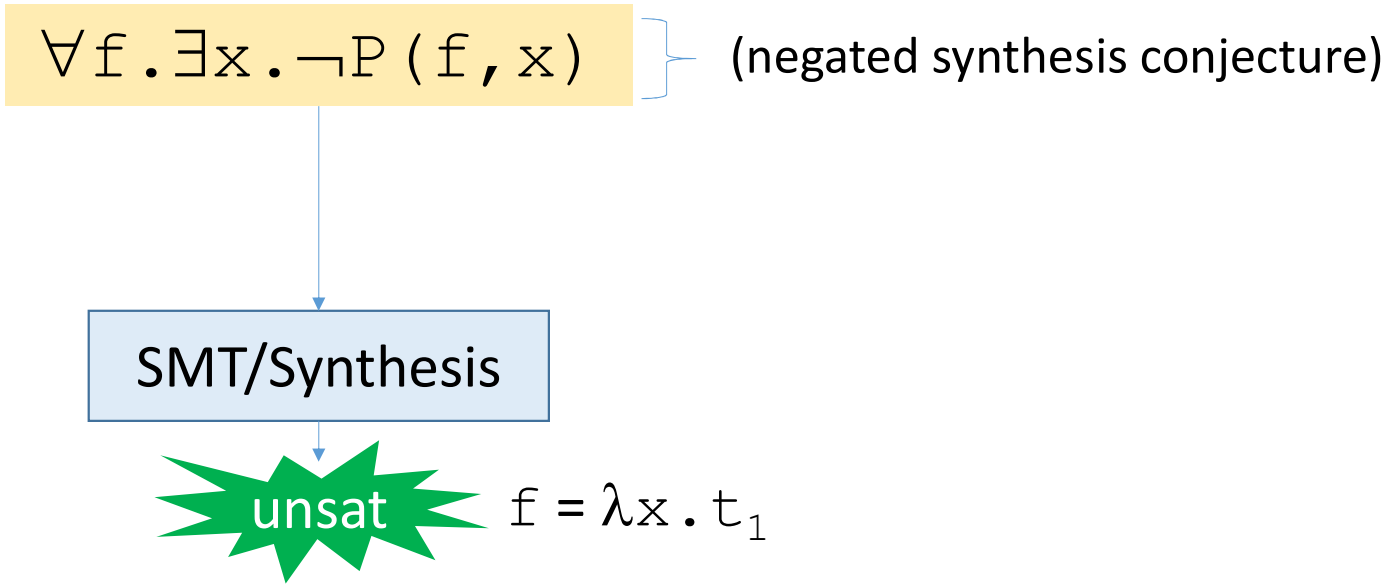
$\exists I.\forall x.(pre(x)\Rightarrow I(x))\wedge(I(x)\Rightarrow post(x))\wedge S'$

$\exists I.\forall xx'.(I(x)\wedge T(x,x'))\Rightarrow I(x')$

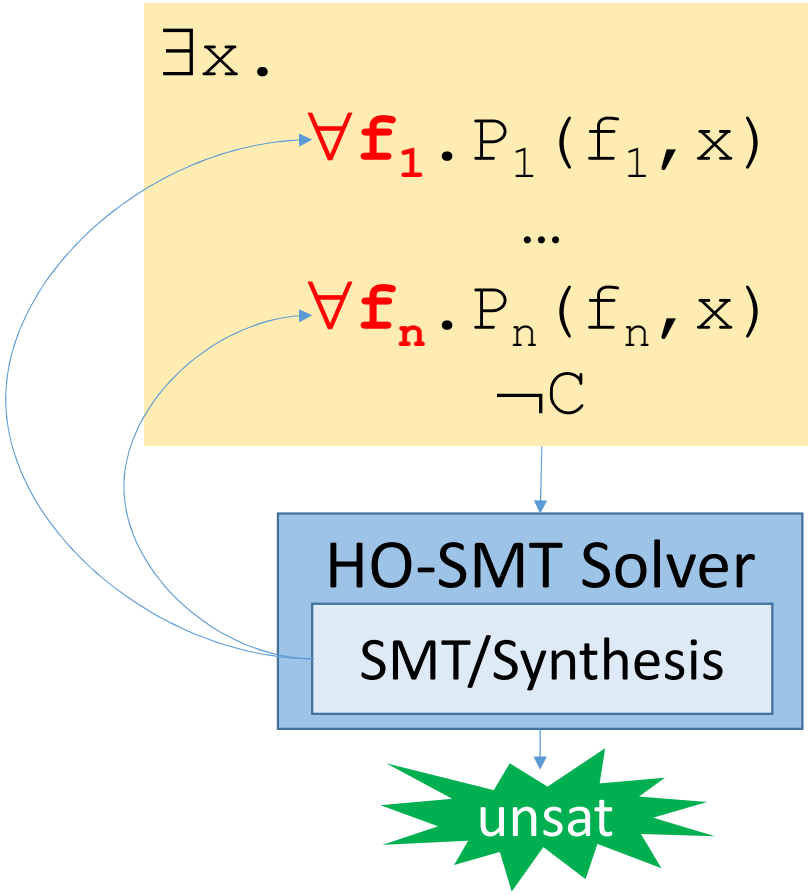


CHALLENGE #3:
How can we combine first-order and second-order techniques for *function* synthesis?

Synthesis for Higher-Order Theorem Proving?



Synthesis for Higher-Order Theorem Proving?



(free function symbols)

(multiple higher-order axioms)

(negated conjecture)

CHALLENGE #4:
Can we use synthesis techniques for *higher order* instantiation?