Math 34 Differential Equations Final Exam
May 10, 2005

SHOW ALL WORK

[12] 1.) Solve: $t^2y' - 3ty = t^5cos(2t)$

Answer 1.) ______________________________

[12] 2.) Solve: $2yy' - \frac{e^{-y^2}}{t^3} = 0$

Answer 2.) ______________________

[12] 3.) Solve $y'' + 2y' + y = tsin(t)$, $y(0) = 0$, $y'(0) = 0$

Answer 3.) ______________________________

[12] 4.) Solve: $y'' + 4y' + 10y = \delta(t - \pi)$, $y(0) = 0$, $y'(0) = 0$

Answer 4.) ______________________________

[12] 5.) Solve: $\mathbf{x}' = \begin{pmatrix} 6 & 1 \\ 12 & 5 \end{pmatrix} \mathbf{x}$. Also describe the behavior of the solution as $t \to \infty$.

Answer 5.) ______________________________

[12] 6.) Use the convolution integral to find the inverse Laplace transform of $\frac{1}{s(s^2+9)}$

Answer 6.) ____________________

[12] 7.) A ball with mass 3 kg is thrown upward with an initial velocity of 10m/sec from the roof of a building 20m high. Neglect air resistance. Find the maximum height above the ground that the ball reaches.

Answer 7.) ______________________________

[12] 8.) A mass weighing 2 kg stretches a spring 4.9m. If the mass is pushed upward an additional 3m and then set in motion with a downward velocity of $3\sqrt{2}$ m/sec, and if there is no damping, determine the position u of the mass at any time t. Find the frequency, period, phase shift and amplitude of the motion.

Answer 8.) ______________________________

[2] 9.) Suppose $y = 2e^t$ is a solution to $ay'' + by' + cy = e^t$. Then a solution to

$ay'' + by' + cy = 5e^t$ is ________________.

[2] 10.) Suppose the following is a direction field in the x_1, x_2 plane for the system $\mathbf{x}' = A\mathbf{x}$ where the eigenvalues of A are $k = 1, -2$. What is the general solution to $\mathbf{x}' = A\mathbf{x}$ (hint: what are the eigenvectors of A?).

11.) Match the following system of differential equation to its direction field (hint: evaluate eigenvectors):

[2] $\mathbf{x}' = \begin{pmatrix} 3 & 3 \\ 4 & -1 \end{pmatrix}$

[2] $\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

Extra problem (can substitute for one of the first 5 problems)

a.) Suppose f_1 and f_2 are solutions to the differential equation $ay'' + by' + cy = 0$. Prove the $c_1 f_1 + c_2 f_2$ is also a solution to $ay'' + by' + cy = 0$.

b.) Suppose f_1 and f_2 are solutions to the differential equation $ay'' + by' + cy^2 = 0$. Prove the $f_1 + f_2$ is NOT a solution to $ay'' + by' + cy^2 = 0$ if neither f_1 nor f_2 is the constant zero function.