

2.3 #22

Short Method:

$$m \frac{dv}{dt} = -mg - \frac{v^2}{1325}$$

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v$$

$$mv \frac{dv}{dx} = -mg - \frac{v^2}{1325}$$

$$\frac{mvdv}{-mg - \frac{v^2}{1325}} = dx$$

$$\text{Let } u = -mg - \frac{v^2}{1325}, \quad du = -\frac{2vdv}{1325},$$

$$-\frac{1325mdu}{2u} = dx$$

$$-\int \frac{1325mdu}{2u} = \int dx$$

$$-\frac{1325m}{2} \ln|u| = x + C$$

$$-\frac{1325m}{2} \ln\left| -mg - \frac{v^2}{1325} \right| = x + C$$

$$-\frac{1325m}{2} \ln\left| mg + \frac{v^2}{1325} \right| = x + C$$

$$x = 0, v = 20$$

$$-\frac{1325m}{2} \ln\left| mg + \frac{20^2}{1325} \right| = C$$

$$> -1325 * .15/2 * \ln(.15 * 9.8 + 20^2/1325);$$

$$-56.84696820$$

$$C = -56.84696820$$

$$v = 0, x = ?$$

$$-\frac{1325m}{2} \ln|mg| - C = x$$

$$> -1325 * .15/2 * \ln(.15 * 9.8);$$

$$-38.28545108$$

$$> -38.28545108 - (-56.84696820);$$

$$18.56151712$$

$$\text{Answer: } 30 + 18.56151563 = 48.562 \text{ m}$$

Long Method:

$$ma = F = -mg - \frac{v^2}{1325}, m = 0.15, g = 9.8$$

$$m \frac{dv}{dt} = -mg - \frac{v^2}{1325}$$

$$\frac{dv}{dt} = -g \left(1 + \frac{v^2}{1325mg}\right)$$

$$\frac{dv}{1 + \frac{v^2}{1325mg}} = -g dt$$

$$\int \frac{dv}{1 + \frac{v^2}{1325mg}} = -g \int dt$$

$$\text{Let } u = \frac{v}{\sqrt{1325mg}}, du = \frac{dv}{\sqrt{1325mg}}, \sqrt{1325mg} du = dv$$

$$\int \frac{\sqrt{1325mg} du}{1+u^2} = -gt + C$$

$$\sqrt{1325mg} \tan^{-1}(u) = -gt + C$$

$$\sqrt{1325mg} \tan^{-1}\left(\frac{v}{\sqrt{1325mg}}\right) = -gt + C$$

$$t = 0, v = 20$$

$$\sqrt{1325mg} \tan^{-1}\left(\frac{20}{\sqrt{1325mg}}\right) = C$$

$$> \text{sqrt}(1325 * .15 * 9.8);$$

$$44.13332075$$

$$> 44.1333 * \text{arctan}(20/44.1333);$$

$$18.77823743$$

$$C = 18.77823743$$

$$v = 0, t = ?$$

$$\sqrt{1325mg} \tan^{-1}\left(\frac{0}{\sqrt{1325mg}}\right) = -gt + C$$

$$0 = -gt + C, gt = C, t = \frac{C}{g}$$

$$> 18.77823743/9.8;$$

$$1.916146677$$

$$t = 1.916146677$$

$$\sqrt{1325mg} \tan^{-1}\left(\frac{v}{\sqrt{1325mg}}\right) = -gt + 18.77823743$$

$$\tan^{-1}\left(\frac{v}{\sqrt{1325mg}}\right) = \frac{-gt+18.77823743}{44.13332075}$$

$$\frac{v}{\sqrt{1325mg}} = \tan\left(\frac{-gt+18.77823743}{44.13332075}\right)$$

$$v = 44.13332075 \tan\left(\frac{-gt+18.77823743}{44.13332075}\right)$$

$$\frac{dx}{dt} = 44.13332075 \tan\left(\frac{-gt+18.77823743}{44.13332075}\right)$$

$$dx = 44.13332075 \tan\left(\frac{-gt+18.77823743}{44.13332075}\right) dt$$

$$dx = 44.13332075 \frac{\sin\left(\frac{-gt+18.77823743}{44.13332075}\right)}{\cos\left(\frac{-gt+18.77823743}{44.13332075}\right)} dt$$

$$\text{Let } u = \cos\left(\frac{-gt+18.77823743}{44.13332075}\right)$$

$$du = -\frac{g}{44.13332075} \sin\left(\frac{-gt+18.77823743}{44.13332075}\right) dt$$

$$dx = -\frac{44.13332075^2}{g} \frac{du}{u}$$

$$\int dx = -\int \frac{44.13332075^2}{g} \frac{du}{u}$$

$$x = -\frac{44.13332075^2}{g} \ln|u| + C$$

$$x = -\frac{44.13332075^2}{g} \ln\left|\cos\left(\frac{-gt+18.77823743}{44.13332075}\right)\right| + C$$

$$t = 0, x = 0 \text{ (or } 30)$$

$$> 44.1333^2 / (-9.8) * \ln(\text{abs}(\cos(18.77823743/44.1333)));$$

$$18.56151563$$

$$C = 18.56151563$$

$$t = 1.91615, x = ?$$

$$x = -\frac{44.13332075^2}{9.8} \ln\left|\cos\left(\frac{-9.8(1.91615)+18.77823743}{44.13332075}\right)\right| + C$$

$$> -44.1333^2 / (9.8) * \ln(\text{abs}(\cos((-9.8 * 1.91615 + 18.77823743) / 44.1333)));$$

$$-0.$$

$$x = 0 + 18.56151563 = 18.56151563$$

$$\text{Answer: } 30 + 18.56151563 = 48.562 \text{ m}$$