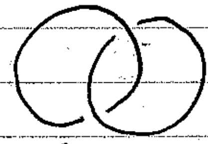
Handout Given

Thursday, ~~March~~ April 1, 2010

Alexander Polynomial of Links

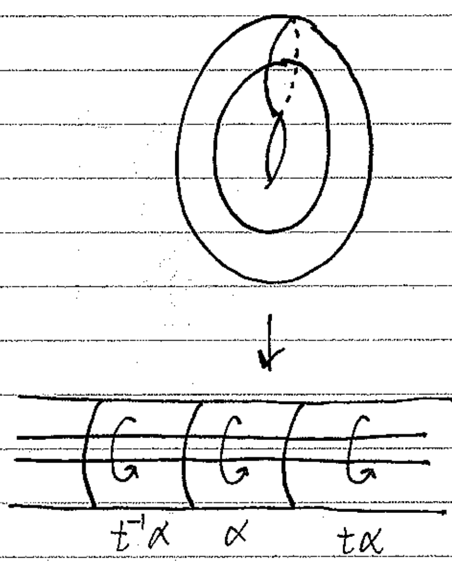
$$
\begin{array}{ll}
\text{universal Abelian cover} & \tilde{X} \\
 & \downarrow \; Aut(\tilde{X}) = \mathbb{Z}\oplus\mathbb{Z} = \mathbb{Z}^2 \\
\infty\text{-cyclic cover} & \hat{X} \\
 & \downarrow \; Aut(\hat{X}) = \mathbb{Z}
\end{array}
$$

Ex 1:



$S^3 \setminus L$

Hopf Link



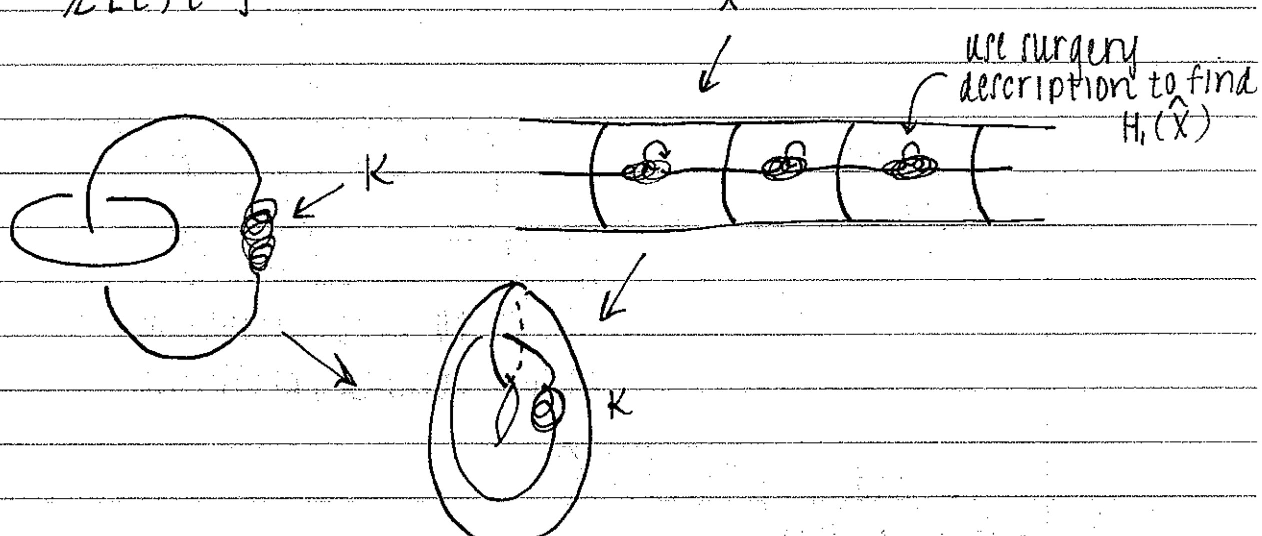
$t^{-1}\alpha = \alpha = t\alpha = t^2\alpha$

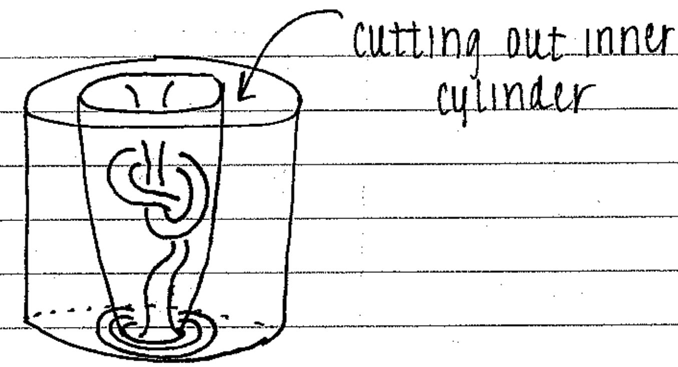
$\alpha(1-t) = 0$

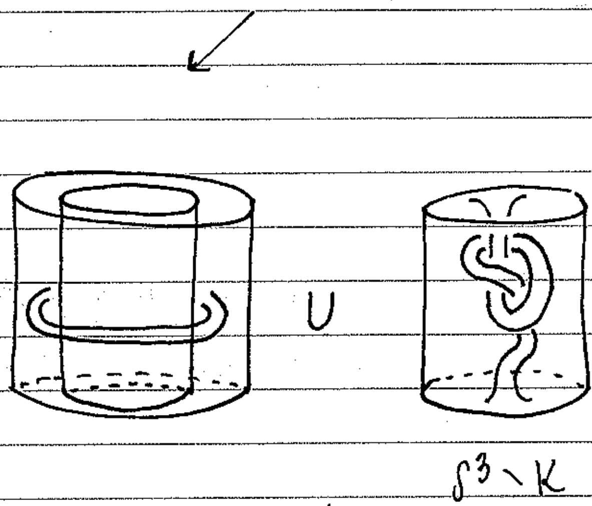
$\Lambda/(1-t)$

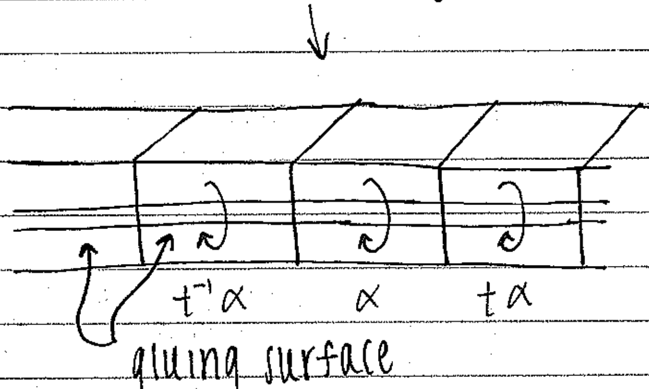
$\mathbb{Z}[t, t^{-1}]$

Ex 4:

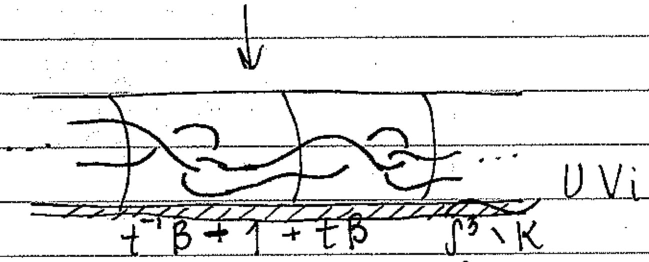








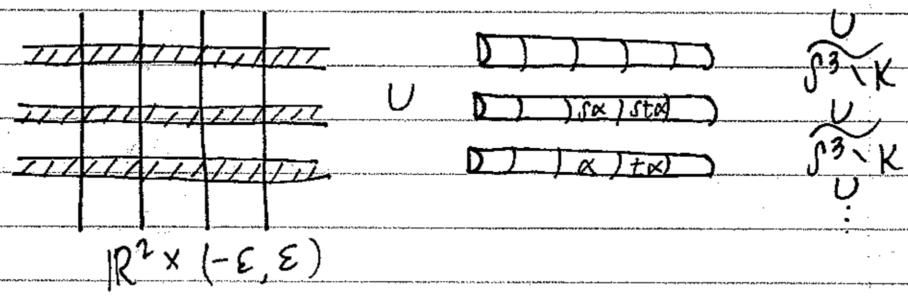
$\tilde{X}$



∞-cyclic cover of $S^3 \setminus K$
= universal abelian cover

see handout

$$\hat{X} = \{\alpha, \beta \mid (1-t)\alpha = 0, \\ (t^{-1} - 1 + t)\beta = 0\}$$



$\mathbb{R}^2 \times (-\varepsilon, \varepsilon)$

$$\Lambda \times \Lambda \big/ (t^{-1} - 1 + t)$$

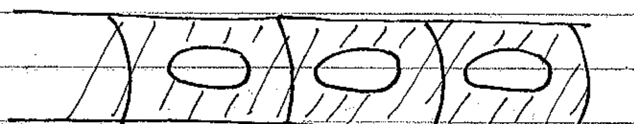
$\mathbb{Z}[s, s^{-1}]$ $\quad$ $\mathbb{Z}[t, t^{-1}]$

Ex 3: Unlink of two components

$$\tilde{X} = \hat{Y}_{-1} \cup N_0 \cup \hat{Y}_0 \cup N_1 \cup \hat{Y}_1 \cup \cdots$$

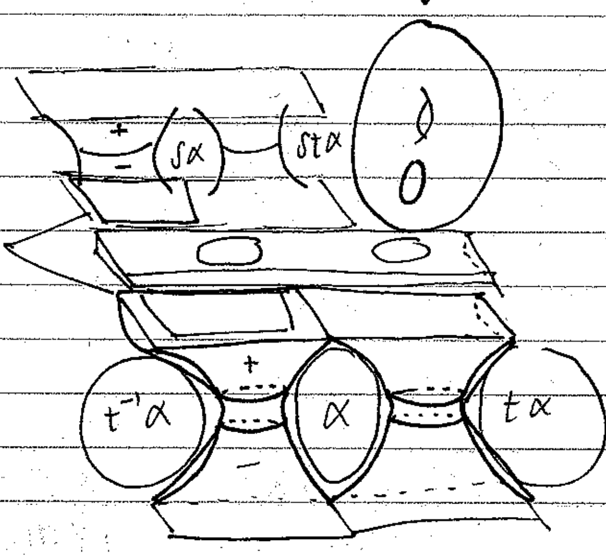
$$\hat{Y}_i = \hat{X} \setminus M \qquad N_0 = M^o \times (-1, 1)$$

$\hat{X}$:



seifert surface

spheres



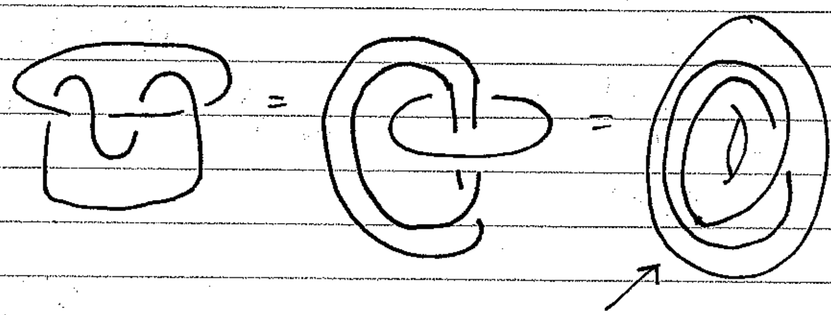
$H_1(\tilde{X}) \cong \Lambda \times \Lambda$

$H_2(O_1^2)$ is interesting

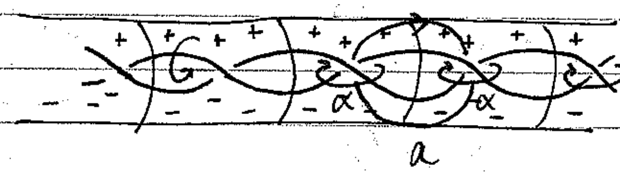
$H_2(O_1^2) = \Lambda \times \Lambda$

s direction   t direction   (spheres in both directions)

EX



$\hat{X}$



$\hat{Y}_i = \hat{X} \setminus M$

$H_1(\hat{Y}_i) = \langle \alpha \rangle$

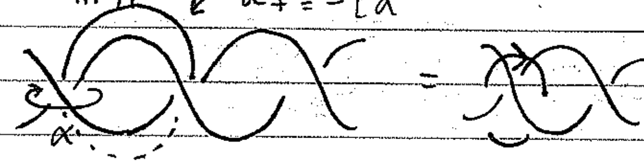
$H_1(N_i) = \langle a \rangle$

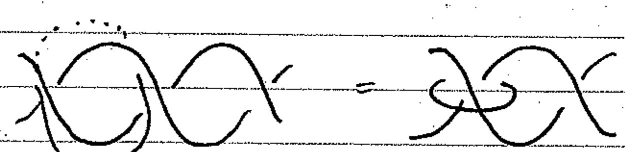
$\tilde{X} = \hat{Y}_{-1} \cup N_0 \cup \underset{\alpha}{\hat{Y}_0} \cup N_1 \cup \underset{s\alpha}{\hat{Y}_1}$

$-st\alpha$ in Y_1

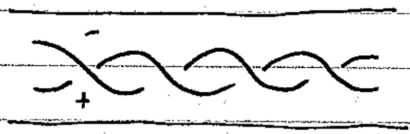
$\underset{\text{in } Y_0}{\alpha = a_-} = a = \underset{\text{in } Y_1}{a_+} = -t\alpha$

$a_+ = -t\alpha$





$\underset{\text{in } Y_0}{\alpha} = \underset{\text{in } Y_1}{-st\alpha}$ $\alpha(1+st)=0$ $\Lambda \times \Lambda / (st+1)$



$-t\alpha \leftarrow a_- = a = a_+ \rightarrow s\alpha$

in Y_0 in Y_1

$-t\alpha = s\alpha$

$(s+t)\alpha$

$H_1 = \Lambda \times \Lambda / (s+t)$

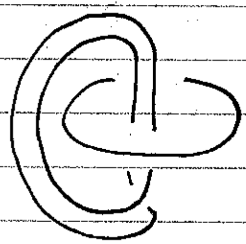
$s+t$

$\downarrow$

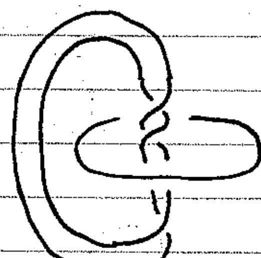
$1+s^{-1}t$

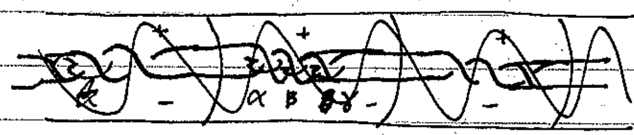
$\downarrow \quad \pm 1 \quad \updownarrow$

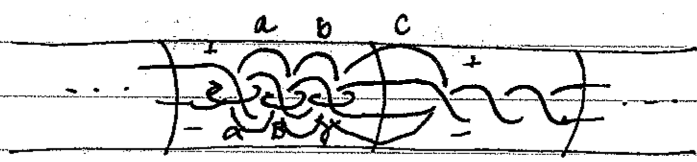
$1+st$



$\cong$







$\hat{Y}_0 \cup_- N_1 \cup_+ Y_1$

$\uparrow \qquad\qquad \uparrow$

$\langle \alpha, \beta, \gamma \rangle \qquad \langle s\alpha, s\beta, s\gamma \rangle$

$\alpha = a^- = a^+ = -s\beta$

$\beta = b^- = b^+ = -s\gamma$

$\gamma = c^- = c^+ = -ts\alpha$

in Y_0 in Y_1

$\langle \alpha, \beta, \gamma \mid \alpha = -s\beta, \cancel{\beta = -s\gamma}, \gamma = -ts\alpha \rangle$ $\Lambda \times \Lambda$ module

$\cong \langle \alpha, \gamma \mid \alpha = s^2\gamma, \gamma = -ts\alpha \rangle \cong \langle \alpha \mid \alpha = -s^3 t\alpha \rangle \quad \Lambda \times \Lambda / (1+s^3t)$