Note T

SEIFERT SURFACE

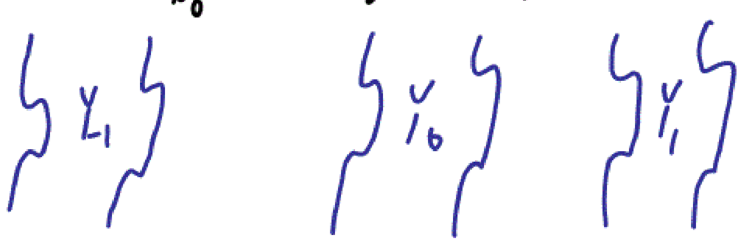
3/1/2010

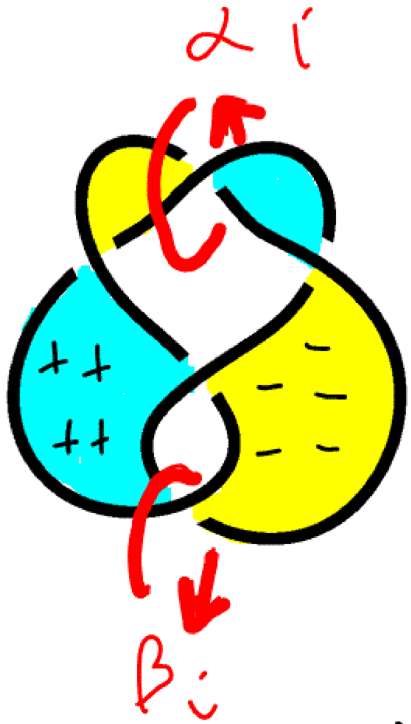
$= M$

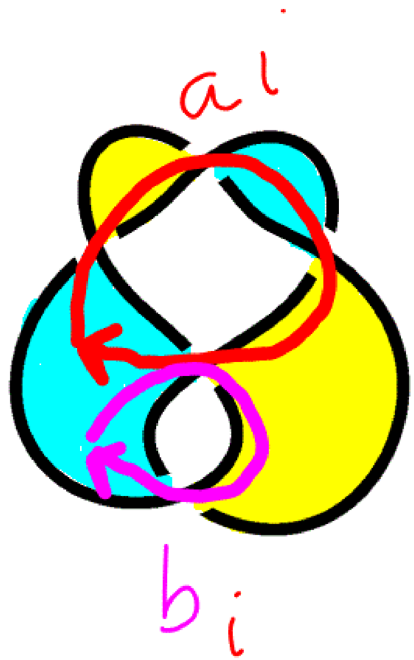
$$Y_i = S^3 \setminus M$$

$$N_i = M^\circ \times (-1,1) = \underset{\substack{\parallel \\ M^\circ \times (-1,0)}}{N_i^-} \cup M \cup \underset{\substack{\parallel \\ M^\circ \times (0,1)}}{N_i^+}$$

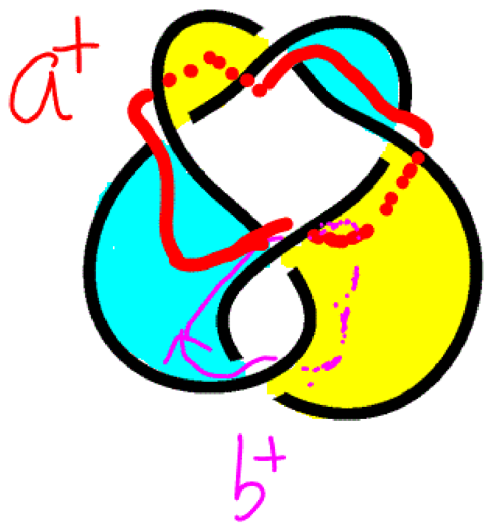
$$\tilde{X} = \widetilde{S^3 \setminus K} = \cdots Y_{-1} \underset{N_0^-}{\cup} N_0 \underset{N_0^+}{\cup} Y_0 \underset{N_1^-}{\cup} Y_1 \cup \cdots$$

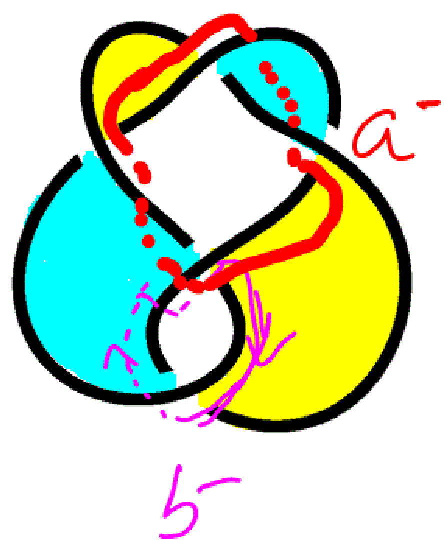


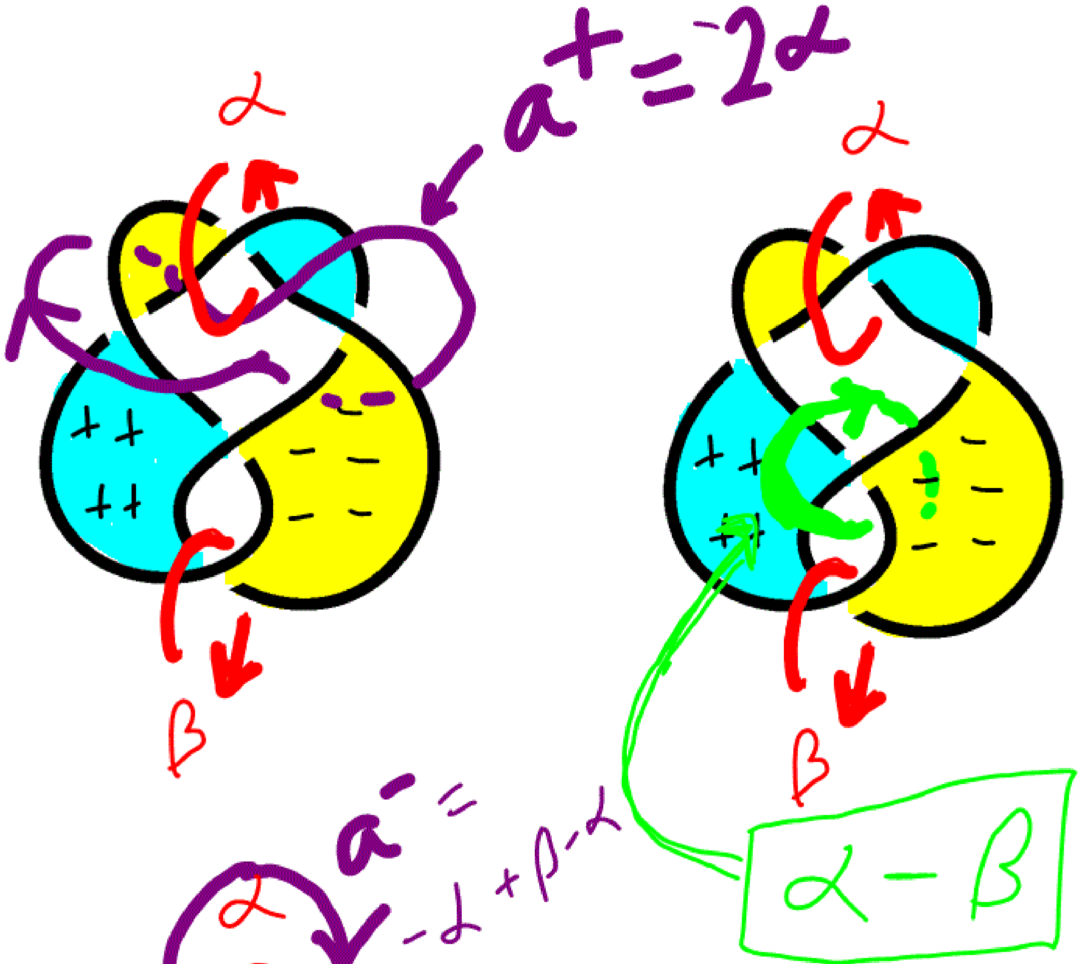


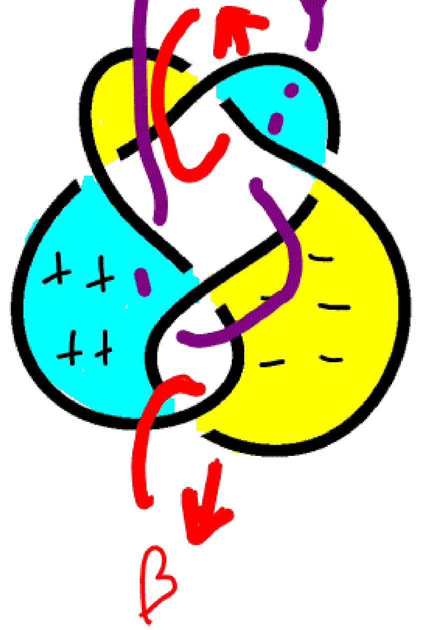


$H_1(Y_i) = (\alpha_i, \beta_i)$ $H_1(N_i) = (a_i, b_i)$









$b_i^+ = \alpha - \beta$

$b_i^- = -\beta$

$a_i^- = -\alpha + \beta - \alpha$
$= \beta - 2\alpha$

$$H_1(\cup Y_i) = (\alpha_i, \beta_i \mid i \in \mathbb{Z})$$

$$a_i^+ = 2\alpha_i$$

$$a_i^- = \beta_{i-1} - 2\alpha_{i-1}$$

$$b_i^+ = \alpha_i - \beta_i$$

$$b_i^- = -\beta_{i-1}$$

$$H_1(\tilde{X}) = (\alpha_i, \beta_i, \text{ for } i \in \mathbb{Z} \mid 2\alpha_i = \beta_{i-1} - 2\alpha_{i-1}, \ \alpha_i - \beta_i = -\beta_{i-1})$$

$$\boxed{\begin{aligned} \beta - 2\alpha &= -2t\alpha \\ -\beta &= t\alpha - t\beta \end{aligned}}$$

We will use t^i to represent i

$$H_1(\tilde{X}) = (t^i\alpha, t^i\beta \mid -2t^i\alpha = t^{i-1}\beta - 2t^{i-1}\alpha, \ t^i\alpha - t^i\beta = -t^{i-1}\beta \text{ for } i \in \mathbb{Z})$$

The Alexander Inv

Let

$$\Lambda = \{ c_{-n} t^{-n} + \dots + c_0 + c_1 t + c_k t^k \mid c_i \in \mathbb{Z},\ n, k \in \mathbb{Z}_+ \cup \{0\} \}$$

= ring of finite Laurent polynomials w/ $\mathbb{Z}$ coeff.

$= \mathbb{Z}[t, t^{-1}]$

$= \mathbb{Z}[J]$ $\quad J = \{ t^i \mid i \in \mathbb{Z} \} \cong \mathbb{Z}$

UNITS : $\pm t^i$

Choose a generator $\tau : \tilde{X} \to \tilde{X}$ of $\mathrm{Aut}(\tilde{X}) \cong \mathbb{Z}$

Let $p(t) = c_{-n} t^{-n} + \dots + c_0 + c_1 t^1 + \dots c_k t^k \in \Lambda$

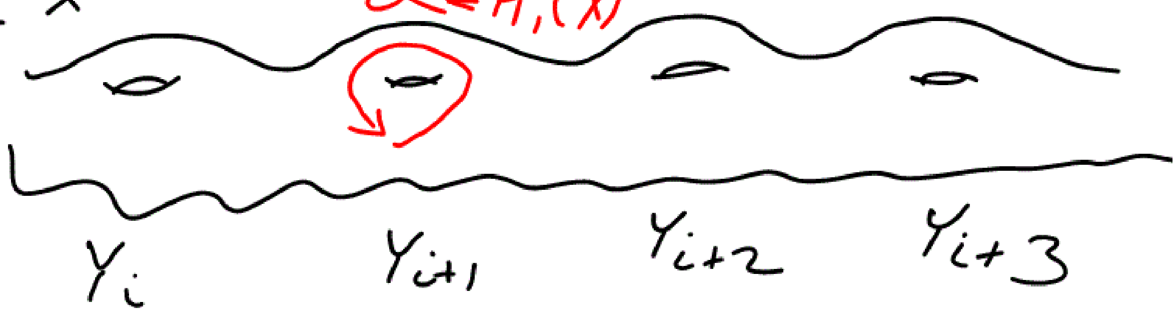
Let $\alpha \in H_i(\tilde{X})$

Define

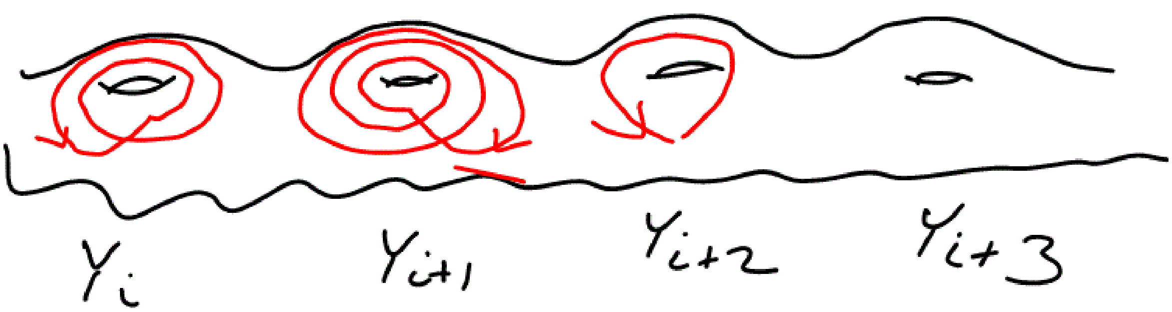
$$p(t)\alpha = c_{-n}\tau_*^{-n}(\alpha) + \dots + c_0\alpha + c_1\tau_*(\alpha) + \dots + c_k\tau_*(\alpha^k) \in H_i(\tilde{X})$$

Ex

$\alpha \in H_1(\tilde{X})$



$\tau \longrightarrow$



$(2t^{-1} + -3 + t)\alpha \in H_1(\tilde{X})$

Note $H_i(\tilde{X})$ is a Λ module

$$H_1(\tilde{X}) = \left(t^i\alpha, t^i\beta \;\middle|\; \begin{array}{l} -2t^i\alpha = t^{i-1}\beta - 2t^{i-1}\alpha \\ t^i\alpha - t^i\beta = -t^{i-1}\beta \\ \text{for } i \in \mathbb{Z} \end{array} \right)$$

as a group

As a Λ-module

$$H_1(\tilde{X}) = \left(\alpha, \beta \;\middle|\; \begin{array}{l} -2t\alpha = \beta - 2\alpha \\ t\alpha - t\beta = -\beta \end{array} \right)$$

$$\beta = -2t\alpha + 2\alpha$$

$$t\alpha + 2t^2\alpha - 2t\alpha = -(-2t\alpha + 2\alpha)$$

$$2t^2\alpha - 3t\alpha + 2\alpha = 0$$

$$H_1(\tilde{X}) = \left(\alpha \mid 2t^2 - 3t\alpha + 2\alpha = 0\right)$$

$$= \Lambda / 2t^2 - 3t + 2$$

Alex invariant

Alexander poly $= 2t^2 - 3t + 2$

or $\pm t^k (2t^2 - 3t + 2)$