Thm: f is differentiable at $\mathbf{a}$ implies f is continuous at $\mathbf{a}$.

Thm: Let $f : \mathbf{R}^n \to \mathbf{R}^m$, $f = (f_1, ..., f_m)$. f is differentiable at $\mathbf{a}$ iff $f_i : \mathbf{R}^n \to \mathbf{R}$ is differentiable at $\mathbf{a}$ for all $i = 1, ..., m$

Thm: If f is differentiable at $\mathbf{a}$ then $\frac{\partial f_i}{\partial x_j}$ exists for all i, j and $Df(\mathbf{a})$ = the Jacobian evaluated at $\mathbf{a}$.

Thm: Let $f : \mathbf{R}^n \to \mathbf{R}^m$, $f = (f_1, ..., f_m)$. If $\frac{\partial f_i}{\partial x_j}$ exists and are continuous in a neighborhood of $\mathbf{a}$ for all i, j, then f is differentiable at $\mathbf{a}$

Ex: Is $f(x, y) = x^2 y$ differentiable at (3, 1).

Find the equation of the tangent plane to $f(x, y) = x^2 y$ at (3, 1).

Estimate f(3.1, .9)

2.4

Thm: If $f, g : \mathbf{R}^n \to \mathbf{R}^m$ is differentiable at $\mathbf{a}$, then $f + g$ is differentiable at $\mathbf{a}$ and $D(f + g) = Df + Dg$.

Thm: Let $c \in \mathbf{R}$. If $f : \mathbf{R}^n \to \mathbf{R}^m$ is differentiable at $\mathbf{a}$, then cf is differentiable at $\mathbf{a}$ and $D(cf) = cDf$.

Thm: If $g : \mathbf{R}^n \to \mathbf{R}^m$ is differentiable at $\mathbf{a}$ and if $f : \mathbf{R}^m \to \mathbf{R}^k$ is differentiable at g($\mathbf{a}$), then $f \circ g$ is differentiable at $\mathbf{a}$ and $D(f \circ g)(\mathbf{a}) = Df(g(\mathbf{a}))Dg(\mathbf{a})$.

Note for the product and quotient rule, f, g are real-valued functions, NOT vector valued.

Thm: If $f, g : \mathbf{R}^n \to \mathbf{R}$ is differentiable at $\mathbf{a}$, then fg is differentiable at $\mathbf{a}$ and $D(fg) = g(\mathbf{a})Df(\mathbf{a}) + f(\mathbf{a})Dg(\mathbf{a})$.

Thm: If $g(\mathbf{a}) \neq 0$ and $f, g : \mathbf{R}^n \to \mathbf{R}$ is differentiable at $\mathbf{a}$, then f/g is differentiable at $\mathbf{a}$ and $D(f/g) = \frac{g(\mathbf{a})Df(\mathbf{a}) - f(\mathbf{a})Dg(\mathbf{a})}{g(\mathbf{a})^2}$.