Defn: f is a function if

$$x = y \Rightarrow [f(x) = f(y)]$$

Examples:
$$f(x) = e^x, \quad f(x) = 4x, \quad f(x) = 0$$
$$f(x) = \ln(x)$$

Prove that $s(x) = \pm\sqrt{x}$ is not a function:
$$s(3) = \pm\sqrt{3} \quad | \quad s(3) = \sqrt{3} \text{ and } -\sqrt{3}$$

Defn: f is an even function if $f(x) = f(-x)$

Examples: $\quad f(x) = \sqrt{4} = 2, \quad h(x) = \cos(x)$
$$g(x) = x^2,$$

Prove that $f(x) = x$ is not an even function:
$$f(-3) = -3 \qquad f(3) = 3$$

Defn: f is an odd function if $f(-x) = -f(x)$

Examples: $\quad f(x) = x^3, \quad g(x) = x$
$$h(x) = \sin(x)$$

Prove that $f(x) = 1$ is not an odd function:
$$f(2) = 1 \qquad f(-2) = 1 \neq -f(2)$$

Defn: f is an increasing function on the interval [a, b] if

$$x < y, \; x, y \in [a, b] \Rightarrow f(x) < f(y)$$

Examples:
$$f(x) = x, \quad h(x) = \ln x$$
$$g(x) = e^x$$

Prove that $f(x) = x^2$ is not an increasing function on the interval [-1, 0]:

$$-.8 < -.2 \quad but \quad .64 > .04$$

Defn: f is an decreasing function on the interval [a, b] if

$$x < y, \; x, y \in [a, b] \Rightarrow f(x) > f(y)$$

Examples:
$$f(x) = -x, \quad g(x) = e^{-x}$$
$$k(x) = -e^x, \quad h(x) = -\ln(x)$$

Prove that $f(x) = x^2$ is not an decreasing function on the interval [0, 1]:
$$0 < 1 \quad but \quad 0^2 < 1^2$$