Final Exam Dec. 13, 2006 SHOW ALL WORK
Math 25 Calculus I Either circle your answers or place on answer line.

1.) Suppose $f(x) = (x-3)^2$ and $g(x) = -x^2 + 8x - 11$.

[3] 1a.) Set up, **but do NOT evaluate**, an integral for the area of the region enclosed by f and g.

[4] 1b.) Set up, **but do NOT evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by the curves f and g about the line $x = 1$ (hint: use cylindrical shells).

[4] 1c.) Set up, **but do NOT evaluate**, an integral for the volume of the solid obtained by rotating the region bounded by the curves f and g about the line $y = -3$ (hint: use washers).

[10] 2.) Find the derivative of $g(x) = \sqrt{ln(x^3 + 1)}$

Answer 3.) ____

[4] 3.) State the mean value theorem.

[4] 4.) Express the following integral as a limit of Riemann sums. Do not evaluate the limit: $\int_0^1 x^2 dx$.

[3] 5.) Is $f : \mathcal{R} \to \mathcal{R}$, $f(x) = x^2$ one-to-one? ________. Explain your answer.

[3] 6.) The domain of $f(x) = ln(x + 3)$ is __________________________.

[10] 7a.) If $y = ln(x)$ find the differential dy and evaluate dy when $x = 1$ and $dx = 0.5$

7b) Find the linearization of $f(x) = ln(x)$ at $x = 1$.

7c.) Use the linearization (or differential) to estimate $ln(1.5)$. Is this an over-estimate or an under-estimate?

[10] 8.) Given $xy = sin(x) + cos(y)$, find y''. You do NOT need to simplify your answer, and you can leave your answer in terms of x and y (y' should not appear in your final answer).

9.) Find the following integrals (SIMPLIFY your answer):

[10] a.) $\int_0^6 \frac{4x+1}{\sqrt{2x+4}}dx =$ ______________________________

[3] b.) $\int \frac{2}{1+x^2}dx =$ ______________________________

10.) Find the following for $f(x) = \frac{5x^2}{x^2+3}$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{30x}{(x^2+3)^2}$ and $f''(x) = \frac{90(1-x)(1+x)}{(x^2+3)^3}$

[1] a.) critical numbers: ______________________

[1] b.) local maximum(s) occur at $x =$ ______________________

[1] c.) local minimum(s) occur at $x =$ ______________________

[1] d.) The global maximum of f on the interval $[0, 5]$ is _______ and occurs at $x =$ ______________

[1] e.) The global minimum of f on the interval $[0, 5]$ is _______ and occurs at $x =$ ______________

[1] f.) Inflection point(s) occur at $x =$ ______________________

[1] g.) f increasing on the intervals ______________________

[1] h.) f decreasing on the intervals ______________________

[1] i.) f is concave up on the intervals ______________________

[1] j.) f is concave down on the intervals______________________

[2] k.) Equation(s) of horizontal asymptote(s)______________________

[4] 6m.) Graph f

Choose 2 out of the following 4 problems. **Clearly indicate which 2 problems you choose.** Each problem is worth 10 points You may do more than 2 problems for up to five points extra credit.

I have chosen the following 2 problems: ______________________________

A.) A rectangular field is bounded on one side by a river and on the other three sides by a fence. Find the dimensions of the rectangular field that will maximize the enclosed area if the fence has total length 100m. How do you know that these dimensions correspond to the enclosed field with maximum area?

Answer A.) ______________________________

B.) At noon, ship A is 100 miles west of ship B. Ship A is sailing west at 40 mph and ship B is sailing north at 30mph. How fast is the distance between the ships changing at 3:00pm?

Answer B.) ____

C.) Use the mean value theorem to show that $f(x) = x^3 + 3x + 5$ has at most one real root.

D.) State the ϵ, δ definition of limit and use this definition to prove $lim_{x \to 2} 3x + 1 = 7$