Exam 2 Nov. 8, 2006
Math 25 Calculus I

SHOW ALL WORK
Either circle your answers or place on answer line.

Sect 4.10

[12] 1.) If $f'(x) = 3x + 8x^{-1} + 2e^x - x^{\frac{5}{2}} - 3$, find $f$.

Answer 1.) $\frac{3}{2}x^2 + 8\ln x + 2e^x - \frac{2}{7}x^{7/2} - 3x + C$

Sect 3.7

[15] 2.) Given $ln(x+y) = 4sin(x)$, find $y''$. You do NOT need to simplify your answer and you can leave your answer in terms of $x$ and $y$ (and only in terms of $x$ and $y$, $y'$ should not appear in your final answer).

$$[\ln(x+y)]' = [4\sin(x)]'$$

$$\left(\frac{1}{x+y}\right)(1+y') = 4\cos(x)$$

$$1 + y' = 4(x+y)\cos(x)$$

$$y' = 4(x+y)\cos(x) - 1$$

$$y'' = 4[(1+y')\cos(x) + (x+y)(-\sin(x))]$$

$$y'' = 4[(1 + 4(x+y)\cos(x) - 1)\cos(x) + (x+y)(-\sin(x))]$$

$$y'' = 4(x+y)[4\cos^2(x) - \sin(x)]$$

Answer 2.)

sect 3.8 [15] 3.) Given $y = x^x$, find $y'$. Simplify your answer.

$$y = x^x$$
$$\ln y = \ln(x^x)$$
$$[\ln y]' = [x \ln(x)]'$$
$$\frac{y'}{y} = x\left(\frac{1}{x}\right) + \ln(x)$$
$$y' = y[1 + \ln(x)]$$

Alternate method:

$$[x^x]' = [e^{\ln x^x}]'$$
$$= [e^{x \ln x}]'$$
$$= e^{x \ln x}\left[x\left(\frac{1}{x}\right) + \ln(x)\right]$$
$$= \cancel{e}^{\ln(x^x)}[1 + \ln(x)]$$

Answer 3.) $y' = x^x[1 + \ln(x)]$

sect 4.4 [15] 4.) $lim_{x\to 0^+}[x^x] =$ 1

$$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\ln x^x} = \lim_{x\to 0^+} e^{x \ln x} = e^{\lim_{x\to 0^+} x \ln x}$$

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$$\lim_{x\to 0^+} x \ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x} \underset{l'H}{=} \lim_{x\to 0^+} \frac{1/x}{-x^{-2}} = \lim_{x\to 0^+} (-x)$$

$$\text{"}\frac{-\infty}{\infty}\text{"} \qquad = 0$$

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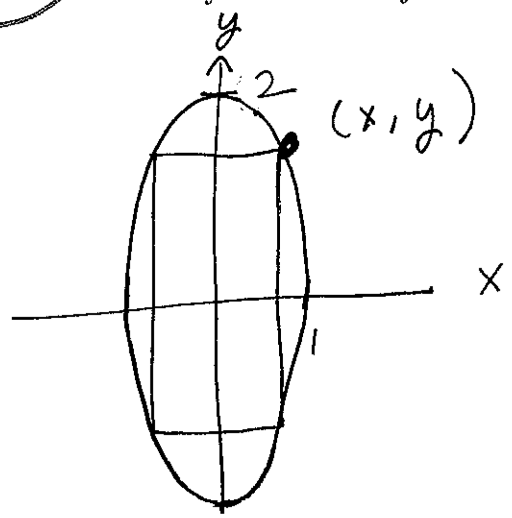
$$\lim_{x\to 0^+} x^x = e^{\lim_{x\to 0^+} x \ln x} = e^0 = 1$$

Sect 4.7

[15] 5.) Find the area of the largest rectangle that can be inscribed in the ellipse, $x^2+\frac{y^2}{4}=1$.

How do you know that your answer is the largest possible area?




width of box $= 2x$

length of box $= 2y$

Area of box $= lw = (2x)(2y)$

$A = 4xy$

Must eliminate one variable

$x^2 + \frac{y^2}{4} = 1 \Rightarrow y^2 = 4(1-x^2)$

$y = 2\sqrt{1-x^2}$

MAXIMIZE $A(x) = 4x(2\sqrt{1-x^2})$ where $x \in [0, 1]$

$A(x) = 8x\sqrt{1-x^2} = 8\sqrt{x^2-x^4}$

$A(x) = 8(x^2-x^4)^{1/2}$

$A'(x) = 4(x^2-x^4)^{-1/2}(2x-4x^3)$

0 or DNE $=$ $A'(x) = \frac{4(2x-4x^3)}{\sqrt{x^2-x^4}} = \frac{8x(1-2x^2)}{x\sqrt{1-x^2}} = \frac{8(1-2x^2)}{\sqrt{1-x^2}}$

$\Rightarrow 1-2x^2 = 0 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \sqrt{1/2}$

$\sqrt{1-x^2} = 0 \Rightarrow x = 1$

| $x$ | $y = 8\sqrt{x^2-x^4}$ |
|---|---|
| 0 | 0 |
| 1 | 0 |
| $\sqrt{\frac{1}{2}}$ | $8(\sqrt{\frac{1}{2}-\frac{1}{4}}) = 4$ |

Answer 5.) Area $= 4$

By EVT, global max at $x = \sqrt{1/2}$. Thus answer is largest possible area.

sect 3.11

[10] 6a.) If $y = x^{\frac{4}{3}}$, find the differential $dy$ and evaluate $dy$ when $x = 8$ and $dx = 0.1$

$$\frac{dy}{dx} = \frac{4}{3} x^{1/3} \Rightarrow \boxed{dy = \frac{4}{3} x^{1/3} dx}$$

when $x = 8$, $dx = 0.1$: $dy = \frac{4}{3}(8)^{1/3}(0.1)$

$$= \frac{\cancel{8}^{4}}{3}\left(\frac{1}{\cancel{10}_{5}}\right) = \frac{4}{15}$$

6b) Find the linearization of $f(x) = x^{\frac{4}{3}}$ at $x = 8$.

$$f'(x) = \frac{4}{3} x^{1/3} \Rightarrow f'(8) = \frac{4}{3}(8)^{1/3} = \frac{8}{3} = \text{slope}$$

point on line: $x = 8$

$f(8) = (8)^{4/3} = 2^4 = 16 \Rightarrow (8, 16)$

$$\frac{y-16}{x-8} = \frac{8}{3} \Rightarrow y = \frac{8}{3}(x-8) + 16 = \frac{8}{3}x + \frac{16}{3}(-4+3)$$

$$y = \frac{8}{3}x - \frac{16}{3}$$

6c.) Use the linearization (or differential) to estimate $(8.1)^{\frac{4}{3}}$

Using linearization near 8, $x^{4/3} \sim \frac{8}{3}x - \frac{16}{3}$

$$(8.1)^{4/3} \sim \frac{8}{3}(8.1) - \frac{16}{3} = \frac{64.8-16}{3} = \frac{48.8}{3}$$

$$= \frac{488}{30} = \frac{244}{15}$$

Alternatively

Let $f(x) = x^{4/3}$

Using differential:

$$f(8.1) = f(8) + f(8.1) - f(8) = f(8) + \Delta f(x)$$

$$\approx f(8) + dy = 8^{4/3} + \frac{4}{15} = 16 + \frac{4}{15} = \frac{244}{15}$$

Sect 4.3/4.5

Note Domain: $[0, \infty)$

$= 0$ or DNE $\to x = 0, 4$

$x^{1/2} = 0$, $x^{1/2} = 8$, $x^{3/2} = 8/3$, $x = 64/9$

7.) Find the following for $f(x) = \frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2} = x^{\frac{3}{2}}(\frac{8-3x^{\frac{1}{2}}}{6})$ (if they exist; if they don't exist, state so). Use this information to graph $f$.

Note $f'(x) = 2x^{\frac{1}{2}} - x = x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0, \text{DNE}$ and $f''(x) = x^{\frac{-1}{2}} - 1 = x^{\frac{-1}{2}}(1 - x^{\frac{1}{2}}) = 0, \text{DNE}$

$x = 1$ $\quad$ $x = 0$

[1] 7a.) critical numbers: 0, 4

[1.5] 7b.) local maximum(s) occur at $x =$ 4

[1.5] 7c.) local minimum(s) occur at $x =$ none

[1.5] 7d.) The global maximum of $f$ on the interval [0, 5] is 8/3 and occurs at $x =$ 4

[1.5] 7e.) The global minimum of $f$ on the interval [0, 5] is 0 and occurs at $x =$ 0

[1.5] 7f.) Inflection point(s) occur at $x =$ 1

[1.5] 7g.) $f$ increasing on the intervals $(0, 4)$

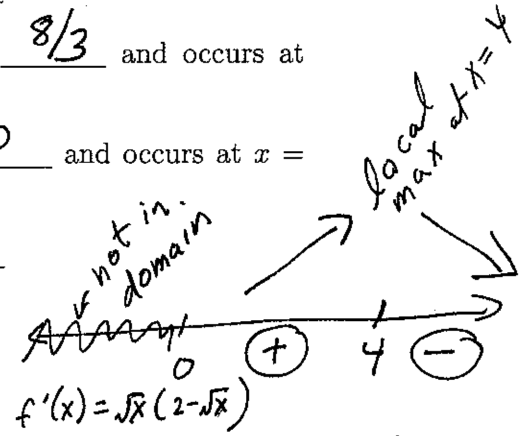
[1.5] 7h.) $f$ decreasing on the intervals $(4, +\infty)$

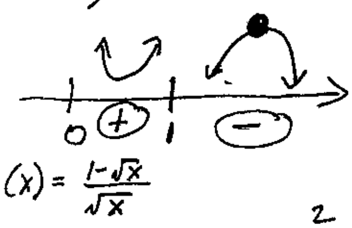
[1.5] 7i.) $f$ is concave up on the intervals $(0, 1)$

[1.5] 7j.) $f$ is concave down on the intervals $(1, \infty)$

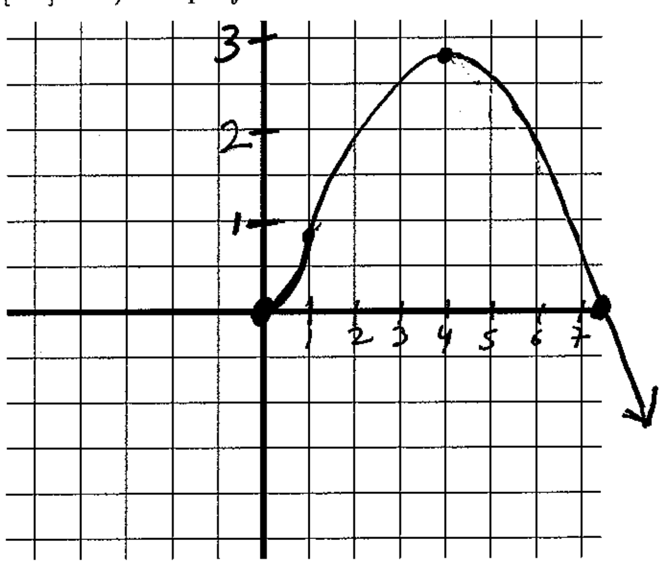
[1] 7k.) Equation(s) of vertical asymptote(s) none

[1] 7l.) Equation(s) of horizontal and/or slant asymptote(s) none




[4.5] 7m.) Graph $f$




| $x$ | $y = \frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2}$ |
|---|---|
| 0 | 0 |
| 64/9 | 0 |
| 4 | $\frac{4}{3}(2^3) - \frac{16}{2} = 8/3$ |
| 1 | $\frac{4}{3} - \frac{1}{2} = 5/6$ |

$$\lim_{x \to +\infty} \left(\frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right) = \lim_{x \to +\infty} x^{3/2}\left(\frac{8 - 3x^{1/2}}{6}\right) = -\infty$$

"$(+\infty)(-\infty)$"