A handle body is a 3-manifold homeomorphic to a connected sum of solid tori.

A Heegard splitting (of genus g) of a 3-manifold M consists of a surface $F = \#_1^g T^2$ which separates M into two handlebodies. I.e. $M = V_1 \cup_f V_2$ where V_i are handlebodies of genus g.

Every closed orientable 3-manifold has a Heegard splitting (use triangulation and let V_1 be thickened 1-skeleton and V_2 corresponds to the dual triangulation).

If F is an orientable surface in orientable 3-manifold M, then F has a collar neighborhood $F \times I \subset M$. F has two sides. Can push F (or portion of F) in one direction.

M is prime if every separating sphere bounds a ball.

M is irreducible if every sphere bounds a ball. M irreducible iff M prime or $M \cong S^2 \times S^1$.

A disjoint union of 2-spheres, S, is independent if no component of $M - S$ is homeomorphic to a punctured sphere (S^3 - disjoint union of balls).

F is properly embedded in M if $F \cap \partial M = \partial F$.

Two surfaces F_1 and F_2 are parallel in M if they are disjoint and $M - (F_1 \cup F_2)$ has a component X of the form $\overline{X} = F_1 \times I$ and $\partial \overline{X} = F_1 \cup F_2$.

A compressing disk for surface F in M^3 is a disk $D \subset M$ such that $D \cap F = \partial D$ and ∂D does not bound a disk in F (∂D is essential in F).

Defn: A surface $F^2 \subset M^3$ without S^2 or D^2 components is incompressible if for each disk $D \subset M$ with $D \cap F = \partial D$, there exists a disk $D' \subset F$ with $\partial D = \partial D'$

Lemma: A closed surface F in a closed 3-manifold with triangulation T can be isotoped so that F is transverse to all simplices of T and for all 3-simplices τ, each component of $F \cap \partial\tau$ is of the form:

Defn: F is a normal surface with respect to T if

1.) F is transverse to all simplices of T.

2.) For all 3-simplices τ, each component of $F \cap \partial\tau$ is of the form:

3.) Each component of $F \cap \tau$ is a disk.

Lemma 3.5: (1.) If F is a disjoint union of independent 2-spheres then F can be taken to be normal.

(2.) If F is a closed incompressible surface in a closed irreducible 3-manifold, then F can be taken to be normal.

Thm 3.6 (Haken) Let M be a compact irreducible 3-manifold. If S is a closed incompressible surface in M and no two components of S are parallel, then S has a finite number of components.