Ex: $f(x) = x^2 - x^3 \overset{\text{optional}}{=} x^3\left(\cancel{\frac{1}{x}}^{0} - 1\right) \sim -x^3$ for large values of x

$lim_{x \to +\infty} x^2 - x^3 = -\infty$

$lim_{x \to -\infty} x^2 - x^3 = +\infty$

Horizontal asymptote(s): none

Ex: $f(x) = x^{\frac{2}{3}} - x^1 \underset{\text{optional}}{=} x\left(\cancel{\frac{1}{x^{1/3}}}_{0} - 1\right) \sim -x$ for large values of x

$lim_{\underline{x \to +\infty}} x^{\frac{2}{3}} - x = -\infty$

$lim_{\underline{x \to -\infty}} x^{\frac{2}{3}} - x = +\infty$

Horizontal asymptote(s): none

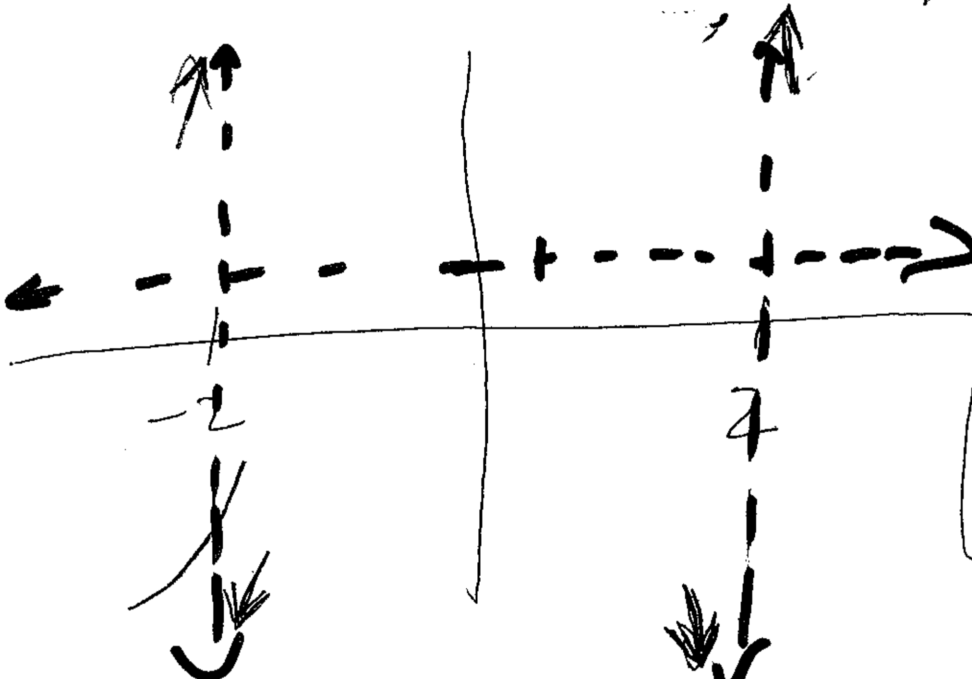
$$f(x) = \frac{x^2}{x^2-4} \sim 1$$ for LARGE x

$\boxed{\text{horizontal asym } y=1}$

As $x \to \pm\infty$, $y \to 1$

Section 3.3:

Motivation: Graph $f(x) = \frac{x^2}{(x-2)(x+2)} = \frac{x^2}{x^2-4}$



$\boxed{\text{vertical asym } x=2,\ x=-2}$

$$\lim_{x \to 2^-} \frac{x^2}{(x-2)(x+2)} = -\infty \qquad \frac{\text{"}4\text{"}}{\text{"}0^-\text{"}\cdot\text{"}4\text{"}}$$

$$\lim_{x \to 2^+} \frac{x^2}{(x-2)(x+2)} = +\infty \qquad \frac{\text{"}+\text{"}}{\text{"}0^+\text{"}\cdot\text{"}+\text{"}}$$

$$\lim_{x \to -2^-} \frac{x^2}{(x-2)(x+2)} = +\infty \qquad \frac{\text{"}+\text{"}}{\text{"}-\text{"}\text{"}0^-\text{"}}$$

$$\lim_{x \to -2^+} \frac{x^2}{(x-2)(x+2)} = -\infty \qquad \frac{\text{"}+\text{"}}{\text{"}-\text{"}\text{"}0^+\text{"}}$$

$\boxed{x \neq 2, -2}$

Find the following for $f(x) = \frac{x^2}{x^2-4} = \frac{x^2}{(x+2)(x-2)}$ (if they exist; if they don't exist, state so). Use this information to graph f.

potential

Note $f'(x) = \frac{-8x}{(x^2-4)^2}$, $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}$

since pts were NOT in domain

f' [1.5] 1a.) critical numbers: $0, 2, -2$

f'' f' [1.5] 1b.) relative maximum(s) occur at $x =$ 0

f'' f' [1.5] 1c.) relative minimum(s) occur at $x =$ none

[1.5] 1d.) The absolute maximum of f on the interval [0, 5] is ______ and occurs at $x =$ ______

[1.5] 1e.) The absolute minimum of f on the interval [0, 5] is ______ and occurs at $x =$ ______

f'' [1.5] 1f.) Inflection point(s) occur at $x =$ none

f' [1.5] 1g.) f increasing on the intervals $(-\infty, -2) \cup (-2, 0)$

$x < -2$

$-2 < x < 0$

f' [1.5] 1h.) f decreasing on the intervals $(0, 2) \cup (2, \infty)$

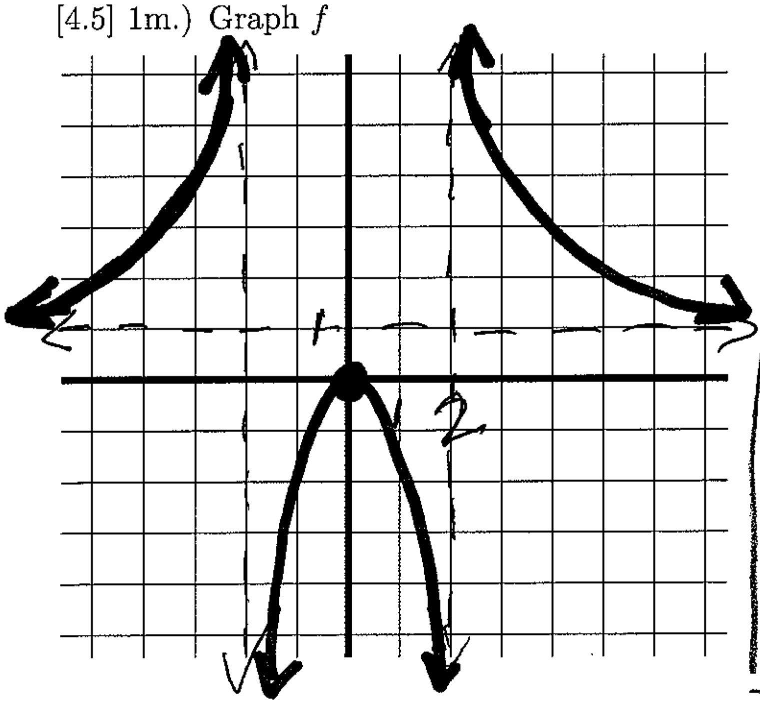
f'' [1.5] 1i.) f is concave up on the intervals $(-\infty, -2) \cup (2, \infty)$

f'' [1.5] 1j.) f is concave down on the intervals $(-2, 2)$

[1.5] 1k.) Equation(s) of vertical asymptote(s) $x = 2$, $x = -2$

[4] 1l.) Equation(s) of horizontal and/or slant asymptote(s) $y = 1$

[4.5] 1m.) Graph f



$$\frac{x^2}{x^2-4} \sim \frac{x^2}{x^2} = 1$$

for large x

$\lim_{x \to 2^+}$ $\quad$ $\lim_{x \to -2^+}$

$\lim_{x \to 2^-}$ $\quad$ $\lim_{x \to -2^-}$

Critical #'s

$$f' = \frac{-8x}{(x^2-4)^2} = 0, \text{ DNE}$$

$$\Rightarrow \boxed{x = 0, 2, -2}$$

$$f'' = \frac{8(3x^2+4)}{(x^2-4)^3} = 0, \text{ DNE}$$

$$\Rightarrow \boxed{x = 2, -2}$$

Note $3x^2+4 > 0$

so $3x^2+4=0$ has no sol'n.

$$f' = \frac{-8x}{(x^2-4)^2}$$

rel max
$x=0$

$+$ $+$ $-$ $-$

$\frac{(-)(-)}{(+)}$ -2 $\frac{(-)(-)}{(+)}$ 0 $\frac{(-)(+)}{(+)}$ 2 $\frac{(-)(+)}{(+)}$

$\frac{(+)}{(+)}$ -2 $\frac{(+)}{(-)}$ 2 $\frac{(+)}{(+)}$

$+$ $-$ $+$

$$f'' = \frac{8(3x^2+4)}{(x^2-4)^3}$$

$x = 2, -2$

Graph
plot important points

x	$y = \frac{x^2}{x^2-4}$
0	0

Find the following for $f(x) = \frac{x^2+3x}{x-1} = \frac{x(x+3)}{x-1}$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{(x-3)(x+1)}{(x-1)^2}$, $f''(x) = \frac{8}{(x-1)^3}$

[1.5] 1a.) critical numbers: $3, -1, \cancel{1}$

[1.5] 1b.) relative maximum(s) occur at $x =$ -1

[1.5] 1c.) relative minimum(s) occur at $x =$ 3

[1.5] 1d.) The absolute maximum of f on the interval [0, 5] is ______ and occurs at $x =$ ______

[1.5] 1e.) The absolute minimum of f on the interval [0, 5] is ______ and occurs at $x =$ ______

[1.5] 1f.) Inflection point(s) occur at $x =$ none

[1.5] 1g.) f increasing on the intervals $(-\infty, -1) \cup (3, \infty)$

[1.5] 1h.) f decreasing on the intervals $(-1, 1) \cup (1, 3)$

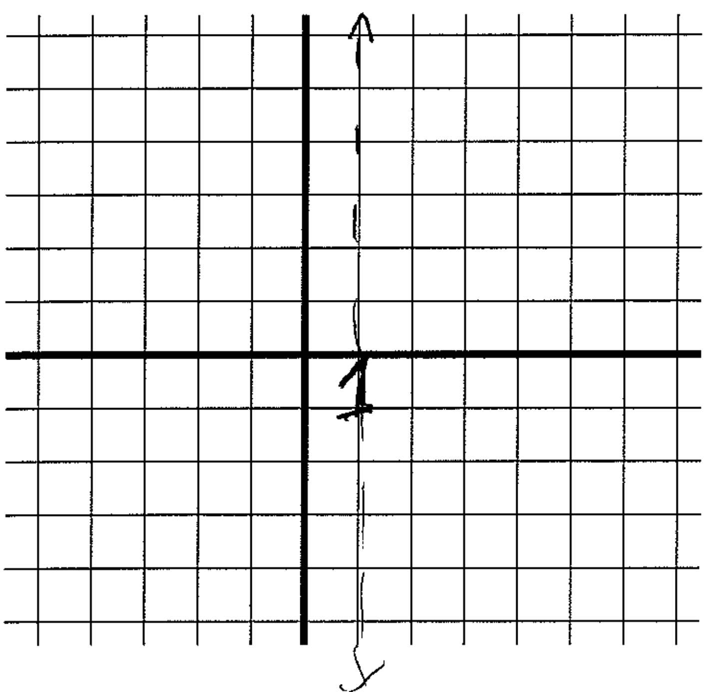
[1.5] 1i.) f is concave up on the intervals $(1, \infty)$

[1.5] 1j.) f is concave down on the intervals $(-\infty, 1)$

[1.5] 1k.) Equation(s) of vertical asymptote(s) $X = 1$

[4] 1l.) Equation(s) of horizontal and/or slant asymptote(s) $y = X + 4$

[4.5] 1m.) Graph f



$$\lim_{X \to 1^-} \frac{X(X+3)}{(X-1)} =$$

$$\lim_{X \to 1^+} \frac{X(X+3)}{(X-1)} =$$

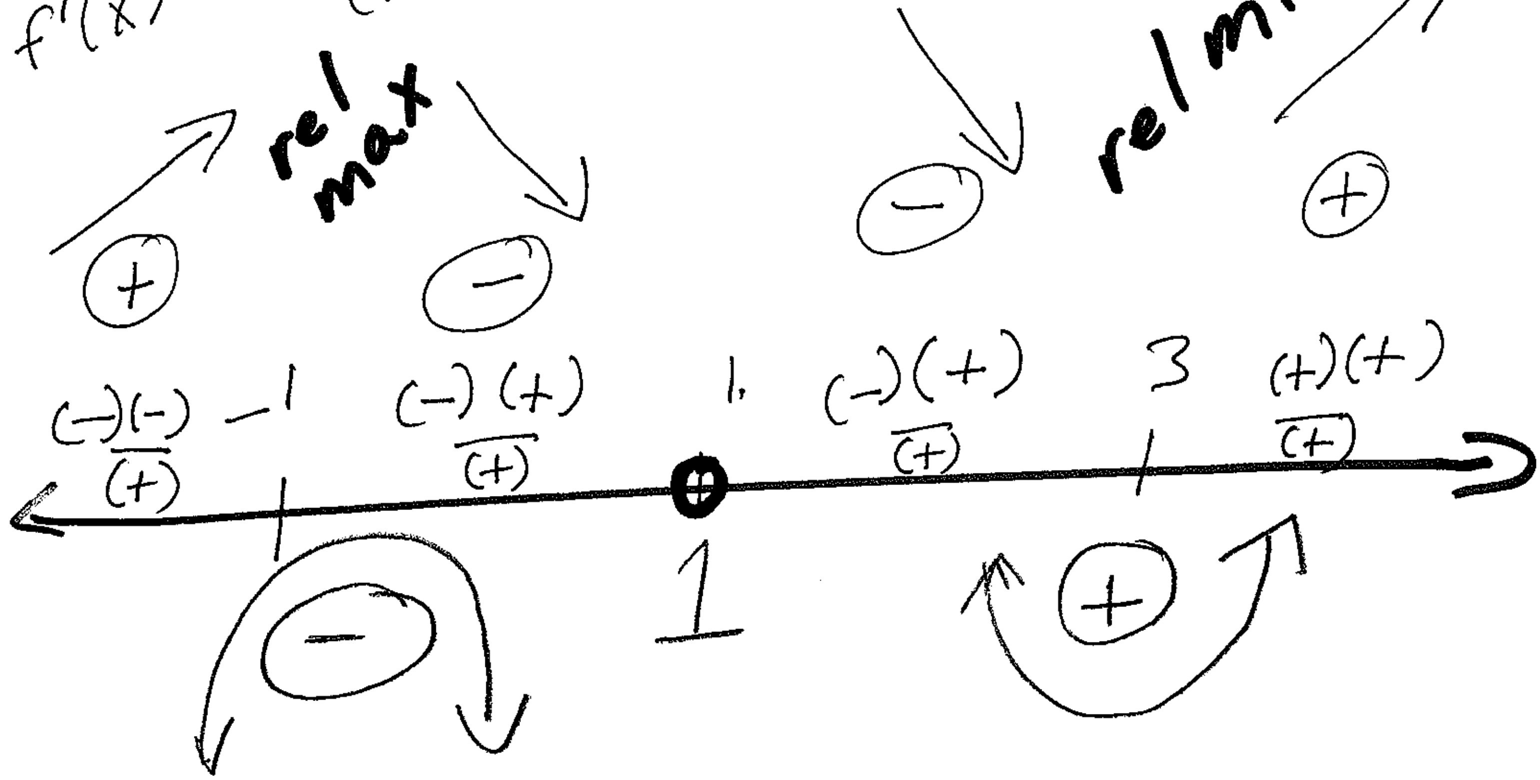
$$f'(x) = \frac{(x-3)(x+1)}{(x-1)^2} = 0, \text{DNE}$$

$$x = 3, -1, 1$$

$$f''(x) = \frac{8}{(x-1)^3} = 0, \text{DNE}$$

$$x = 1$$

$$f'(x) = \frac{(x-3)(x+1)}{(x-1)^2}$$



$$f''(x) = \frac{8}{(x-1)^3}$$

$$\lim_{x\to+\infty} \frac{x^{2}+3x}{x-1} = +\infty$$

$$\lim_{x\to-\infty} \frac{x^2+3x}{x-1} = -\infty$$

$$\frac{(+)}{(-)}$$

no horizontal asymptote
But we do have a slant asymptote

$$\begin{array}{r} x+4 \ r\ 4 \\ (x-1)\overline{\big)\, x^2+3x} \\ \underline{x^2-x} \\ 4x \\ \underline{4x-4} \\ +4 \end{array}$$

$$\frac{x^2+3x}{x-1}$$

$$= \underline{x+4} + \frac{4}{x-1} \to 0$$

$\approx x+4$ for large x

check : $\dfrac{(x+4)(x-1)+4}{x-1}$

$$= \frac{x^2+3x-4+4}{x-1}$$