5.5) Integration by sub

$$\left[\ln|x^2-x|\right]' = \frac{2x-1}{x^2-x}$$

$$\Longrightarrow$$

$$\int \frac{(2x-1)\,dx}{x^2-x} = \ln|x^2-x| + C$$

To recognizer an derivative/ anti derivative use u-substition

~~Let u~~

Let $u = x^2 - x$

$$\frac{du}{\cancel{dx}} = (2x-1)\,\cancel{dx}$$

$$du = (2x-1)\,dx$$

$$\int \frac{(2x-1)\,dx}{x^2-x} = \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$= \boxed{\ln|x^2-x| + C}$$

Chose a u which simplified the problem

②

1) $\int \underline{2x}\, e^{x^2}\, \underline{dx}$

Let $u = x^2$

$du = 2x\,dx$

$$\int \underline{2x}\, e^{x^2}\, \underline{dx} = \int e^u\, du$$

$$= e^u + C = \boxed{e^{x^2} + C}$$

2) $\int_0^1 3x^2\sqrt{x^3+1}\,dx$

Let $u = x^3+1$

$du = 3x^2\,dx$

$\int \underbrace{3x^2}\sqrt{x^3+1}\,\underbrace{dx} =$

$= \int \sqrt{u}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(x^3+1)^{3/2} + C$

cont

$$\int_0^1 3x^2\sqrt{x^3+1}\,dx$$

$$= \frac{2}{3}(x^3+1)^{3/2}\Big|_0^1$$

$$= \boxed{\frac{2}{3}(2)^{3/2} - \frac{2}{3}}$$

Alternatively,

$$\Rightarrow$$

Check $\left[\frac{2}{3}(x^3+1)^{3/2}\right]' = 3x^2\sqrt{x^3+1}$

$$\int_0^1 3x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3+1 \Rightarrow du = 3x^2\,dx$

$x = 0 \Rightarrow u = 0^3+1 = 1$

$x = 1 \Rightarrow u = 1^3+1 = 2$

$$\int_0^1 \underline{3x^2}\sqrt{x^3+1}\,\underline{dx}$$

$$= \int_1^2 \sqrt{u}\,du$$

$$= \frac{2}{3}u^{3/2}\Big|_1^2$$

$$= \boxed{\frac{2}{3}(2)^{3/2} - \frac{2}{3}}$$

$$\frac{1}{2}\int_1^3 \frac{2x\,dx}{x^2+4}$$

Let $u = x^2+4$

$du = 2x\,dx$

(or $\frac{du}{2} = x\,dx$)

$x=1 \Rightarrow u = 1^2+4 = 5$

$x=3 \Rightarrow u = 3^2+4 = 13$

$$\frac{1}{2}\int_1^3 \frac{2x\,dx}{x^2+4} = \frac{1}{2}\int_5^{13}\frac{du}{u}$$

$$= \frac{1}{2}\ln|u|\Big|_5^{13} = \frac{1}{2}(\ln 13 - \ln 5)$$

$$= \frac{1}{2}\ln\frac{13}{5} = \ln\sqrt{\frac{13}{5}}$$

4) FYI $\displaystyle\int_0^8 x\sqrt{1+x}\,dx$

Let $u = 1 + x \Rightarrow x = u - 1$

$du = dx$

$$\int x\sqrt{u}\,du$$

$$= \int (u-1)\,u^{1/2}\,du$$

$$= \int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$$

$$\int_0^8 x\sqrt{1+x}\,dx$$

$$= \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2}\Big|_0^8$$

$$= \boxed{\frac{2}{5}(3^5) - \frac{2}{3}(}$$

5) $\int \sqrt{x}\,(x^2-1)\,dx$

$$= \int x^{1/2}(x^2-1)\,dx$$

$$= \int \left(x^{5/2} - x^{1/2}\right) dx$$

simplify

$$= \frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2} + C$$

6) $\int \cos^3 x \, dx$

$= \int \cos^2 x \cdot \cos x \, dx$

$= \int (1 - \sin^2 x) \, \underline{\cos x \, dx}$

Let $u = \sin x$

$du = \cos x \, dx$

$= \int (1 - u^2) \, du$

$= u - \frac{1}{3} u^3 + C$

$= \boxed{\sin x - \frac{1}{3} \sin^3 x + C}$

7) $\int_0^{\pi} \cos^3 x \, dx$

$= \sin x - \frac{1}{3} \sin^3 x \Big|_0^{\pi}$

$= \sin \pi - \frac{1}{3} \sin^3 x - \left(\sin(0) - \frac{1}{3}\sin^3(0)\right.$

$= 0$

7) $\int_0^{\pi} \cos^3 x \, dx$

$= \int_0^{\pi} (1-\sin^2 x) \cos x \, dx$

Let $u = \sin x \Rightarrow$ ~~$du = \cos x \, d$~~
$du = \cos x \, dx$

$\left.\begin{matrix} x = 0 \\ x = \pi \end{matrix}\right\} \begin{matrix} \Rightarrow u = \sin(0) = 0 \\ \Rightarrow u = \sin(\pi) = 0 \end{matrix}$

$= \int_0^0 (1-u^2) \, du$

$= 0$

Method 3

$$\int_0^{\pi} \cos^3 x \, dx = 0$$