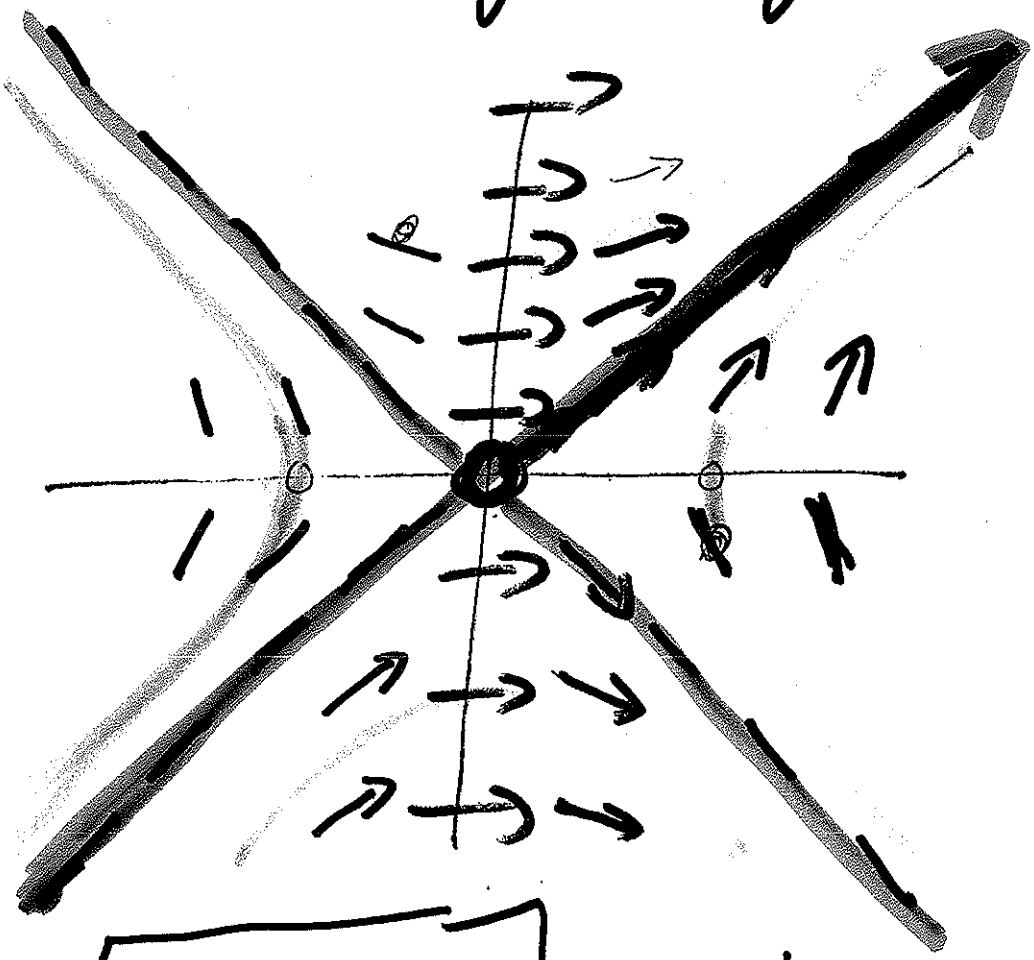
$$\frac{dy}{dx} = \frac{x}{y}$$

$y = 1 : y' = x$

$y = -1 : y' = -x$



$x = 0 \Rightarrow$ slope $= \frac{0}{y} = 0$

$x = 1 \Rightarrow$ slope $= \frac{1}{y}$

$x = y \Rightarrow$ slope $= \frac{x}{x} = 1$

$y = -x$ slope $= \frac{x}{-x} = -1$

$x = -1$ slope $= \frac{-1}{y}$

some Sol'ns: $y \neq 0$ $\begin{cases} y = x \Rightarrow \frac{dy}{dx} = 1 = \frac{x}{y} \\ y = -x \end{cases}$

$y = 1$: slope $= x/1 = x$

$y = -1$: slope $= \frac{x}{-1}$

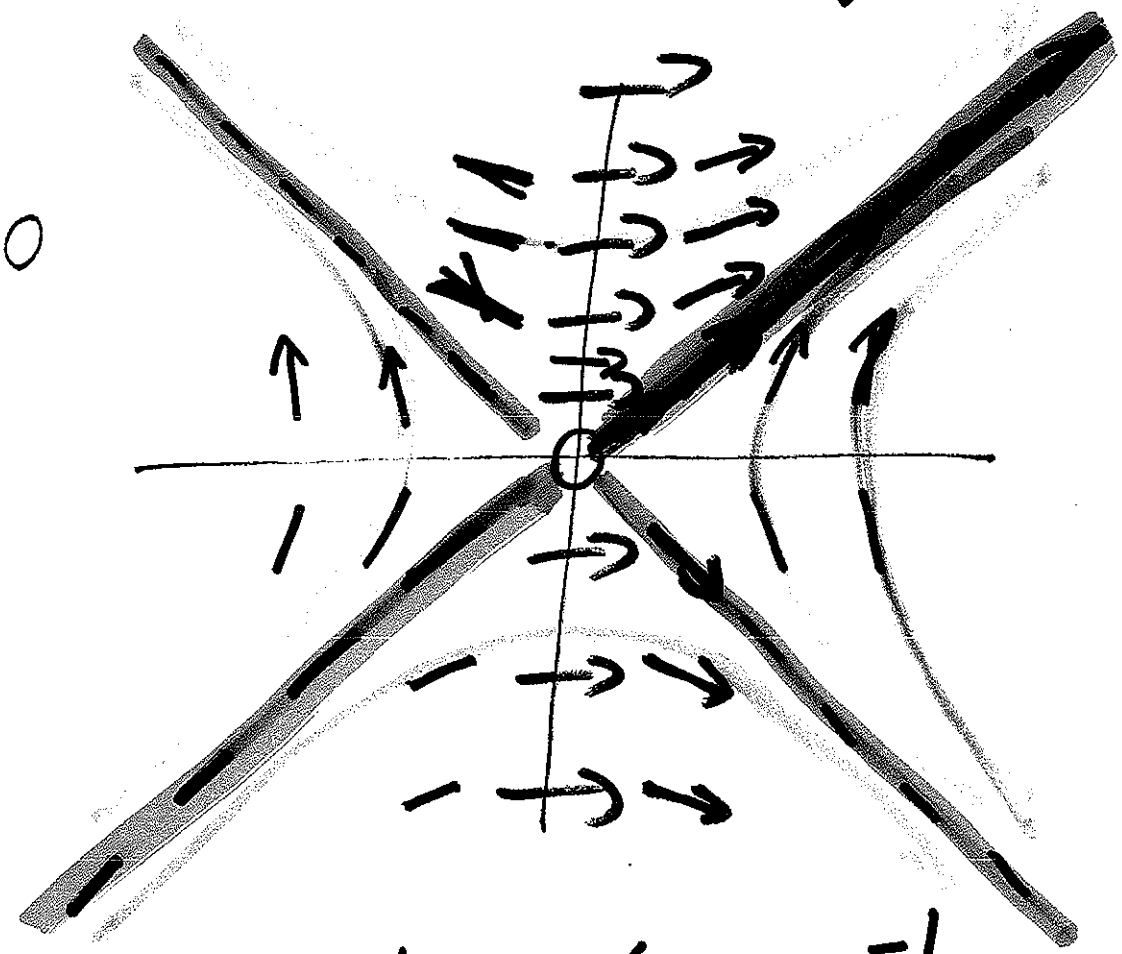
$\frac{dy}{dx} = \frac{x}{y}$

$x = -y$

$y' = \frac{-y}{+y} = -1$

$x = 0 \Rightarrow$ slope $= \frac{0}{y} = 0$

$x = 1 \Rightarrow$ slope $= \frac{1}{y}$



$x = -1$: slope $= \frac{-1}{y}$

$x = y \Rightarrow$ slope $= \frac{x}{x} = 1$

Note $x = y$ is a soln to $y' = \frac{x}{y}$

$y = x \Rightarrow y' = 1 = \frac{x}{x} = \frac{x}{y}$ since $y = x$

Note $y = -x$ is a soln to $y' = \frac{x}{y}$

since $y' = -1 = \frac{-y}{y} = \frac{x}{y}$

8.4) separable diff eqns

get y's side on one side

$$y\frac{dy}{dx}dx = \frac{x}{y}\cdot y\,dx$$

put x's on the other side

$$\int y\,dy = \int x\,dx$$

Note dx & dy are on top

need height · width

to integrate

height · dx

height · dy

★★★★★★★★★★

correct but not needed

$$\frac{y^2}{2} + h = \frac{x^2}{2} + C$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + (C - h)$$

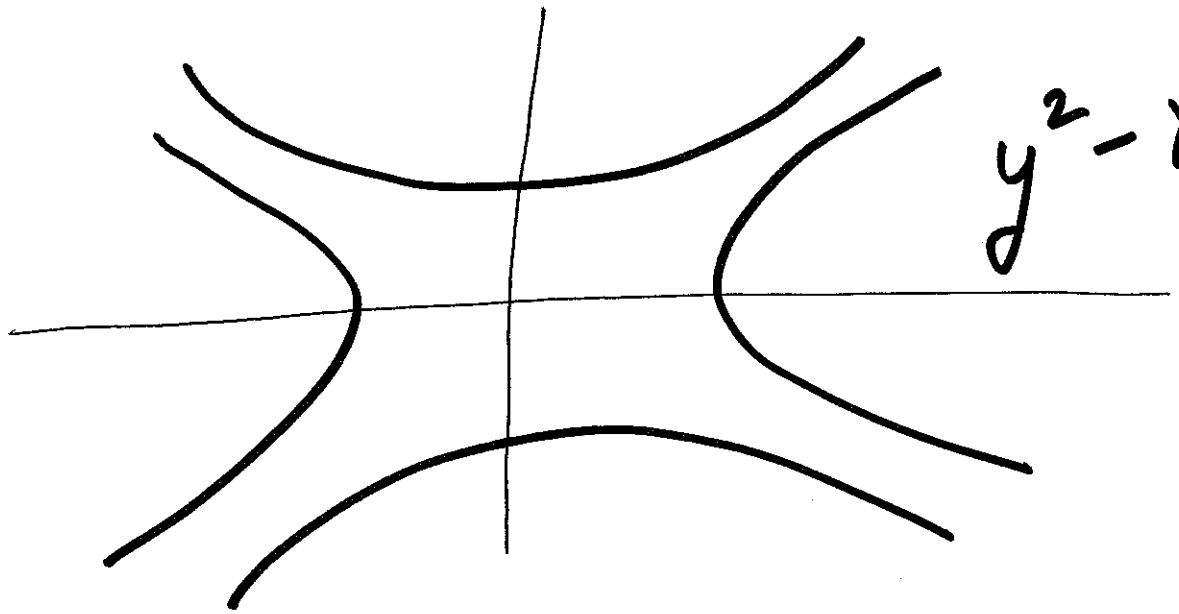
just a constant

$$\int y\,dy = \int x\,dx$$

$$2\left(\frac{y^2}{2}\right) = \left(\frac{x^2}{2} + C\right)2$$

$$\boxed{y^2 = x^2 + C}$$

note this general soln is equivalent to $y^2 = x^2 + 2C$



Ex 2: $y' = ky$ k is a constant

$$\frac{dy}{y\,dt}\,dt = \frac{ky}{y}\,dt$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| = kt + C$$

$$e^{\ln|y|} = e^{kt+C}$$

$$|y| = e^{kt}e^{C}$$

$$y = \left(\pm e^{c}\right) e^{kt}$$

$$\boxed{y = Ce^{kt}}$$

Ex 3: $\frac{2y'}{2} + \frac{10y}{2} = \frac{16}{2}$

Need height, width

$$\frac{dy}{dt} + 5y = 8$$

$$\frac{dt}{8-5y}\left(\frac{dy}{dt}\right) = \frac{(8-5y)}{8-5y}\,dt$$

$$\int \frac{dy}{8-5y} = \int dt$$

$$-\frac{1}{5}\ln|8-5y| = t + C$$

or use u-substitution

$$\ln|8-5y| = -5t + C$$

$$e^{\ln|8-5y|} = e^{-5t+C}$$

$$|8-5y| = e^{-5t}\, e^{C}$$

$$8-5y = (\pm e^{C})e^{-5t}$$

$$8-5y = Ce^{-5t}$$

$$\frac{-5y}{-5} = \frac{Ce^{-5t}}{-5} - \frac{8}{-5}$$

$$\boxed{y = Ce^{-5t} + \frac{8}{5}}$$