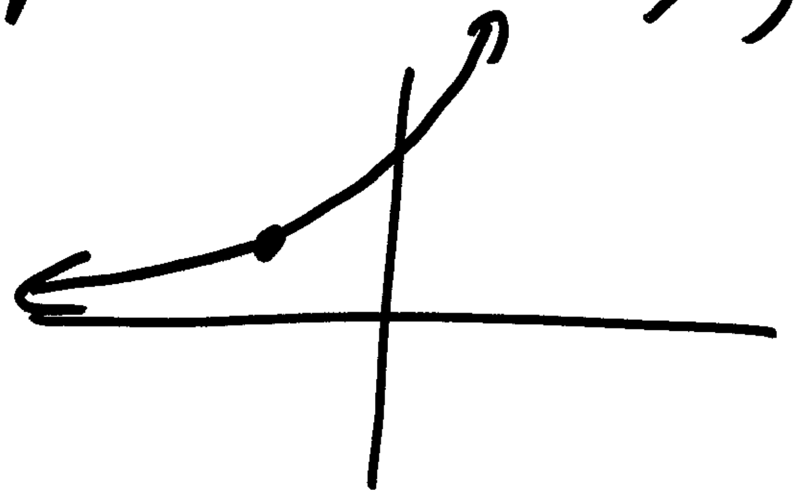
To specify which $y = ce^{kx}$

Need to choose a point (x_0, y_0)



Initial value problem

Solve $\frac{dy}{dx} = ky$ where $y(x_0) = y_0$

(ie (x_0, y_0) lies on graph of the sol'n)

There exists a unique sol'n to this I.V.P.

$y' = ky \iff y = ce^{kx}$

Fix k

(x_0, y_0) lies on $y = Ce^{kx}$ (pre-calculus)

IVP (calculus)

$$y_0 = Ce^{kx_0}$$

$$\Rightarrow C = y_0 / e^{kx_0}$$

Solve Initial Value Problem

$$\frac{dy}{dt} = 5y \quad \text{where } y(0) = 3$$

$\Downarrow$

$$y = Ce^{5t}$$

$$3 = Ce^{0} \Rightarrow C = 3$$

$$\boxed{y = 3e^{5t}}$$

Check $y' = 15e^{5t}$
$5y = 15e^{5t}$ ✓

Suppose invest \$1000 at a rate of 3% /yr compounded <u>continuously</u>

a) How much after 1 yr

b) When will you double your money?

$$\frac{dP}{dt} = 0.03P$$

$$\Rightarrow P = Ce^{0.03t}$$

$$t=0, \quad 1000 = Ce^{0}$$

$$\Rightarrow C = 1000$$

$$P = 1000e^{0.03t}$$

a) $P(1) = 1000e^{0.03(1)} = 1030.45$

b) Find t when $P = 2000$

$$2000 = 1000e^{0.03t}$$

$$\ln 2 = \ln e^{0.03t}$$

$$\ln 2 = 0.03t$$

$$t = \frac{\ln 2}{0.03}$$

$$= \frac{0.69\ldots}{0.03}$$

$$= 23.1\ldots$$

$$\boxed{\ln 2 = 0.69\ldots}$$

<u>Doubling time</u> = generation time

= the time it takes to double initial amount

$$P(t) = P_0 e^{kt}$$

$$P(0) = P_0$$

Find t when $P(t) = 2P_0$

$$2\cancel{P_0} = \cancel{P_0} e^{kt}$$

$$2 = e^{kt}$$

$$\ln 2 = \ln e^{kt}$$

$$\ln 2 = kt \Rightarrow t = \frac{\ln 2}{k} \approx \frac{0.69}{K}$$

t	$z = f(t)$
1	1
10	100
100	10,000

$\Rightarrow$

$x = \log t$	$y = \log z$
0	0
1	2
2	4

~~log~~ $\log t = \log_{10} t$

$\log(1) = 0$ since $10^0 = 1$

$\log(10) = 1$ " $10^1 = 10$

$\log(100) = 2$ " $10^2 = 100$

$$y = 2x \Rightarrow z = t^2$$

log-log plots

$z = f(t)$

determine f

Let $x = \log t$; $y = \log z$

Suppose $y = mx + b$

$\log z = m \log t + b$

$$y = mx + b$$

$$\log z = m \log t + b$$

$$10^{\log z} = 10^{m \log t + b}$$

$$z = \left(10^{m \log t}\right)\left(10^{b}\right)$$

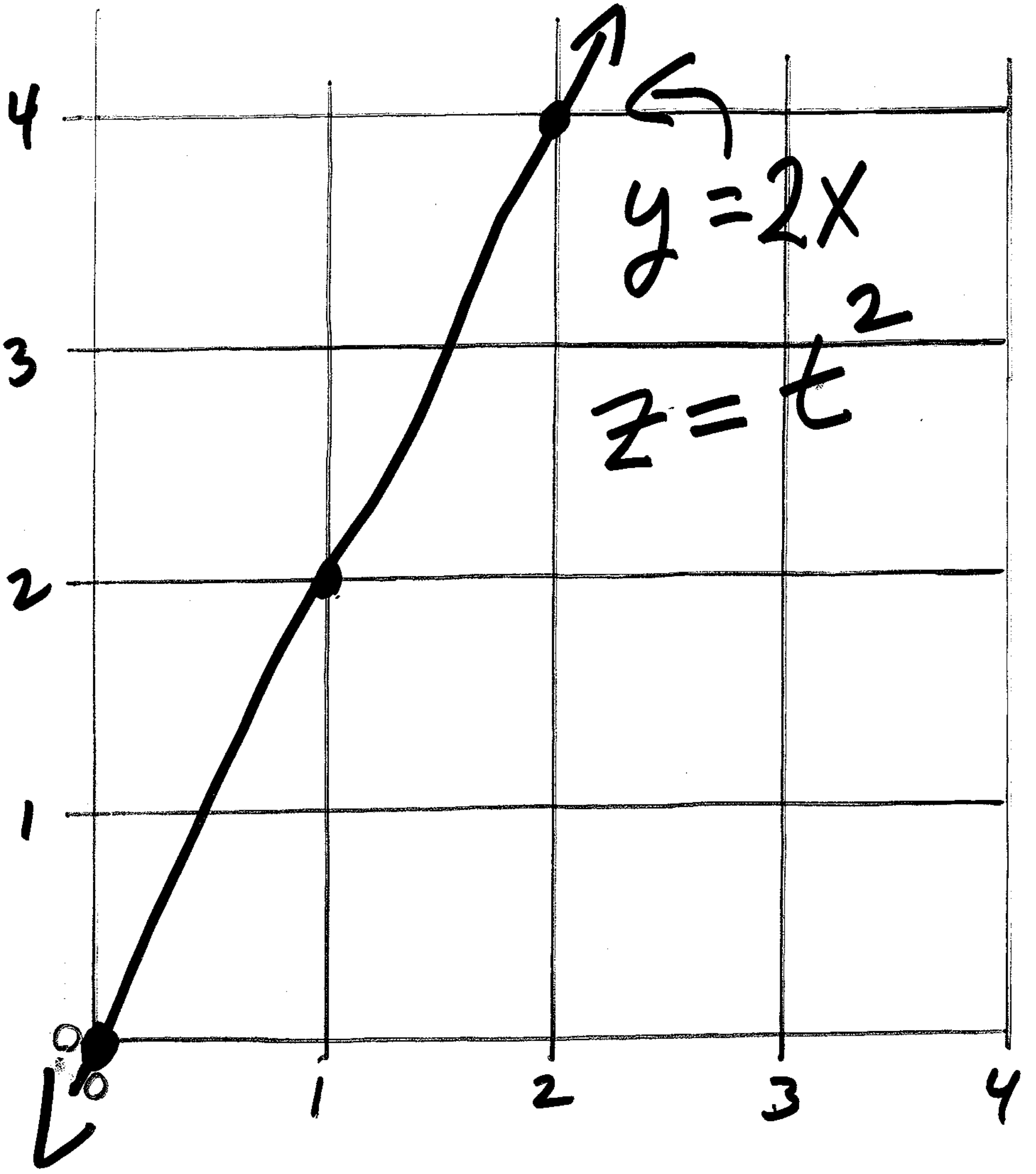
$$z = \left(10^{\log t^{m}}\right)\left(10^{b}\right)$$

$$z = \left(10^{b}\right) t^{m}$$

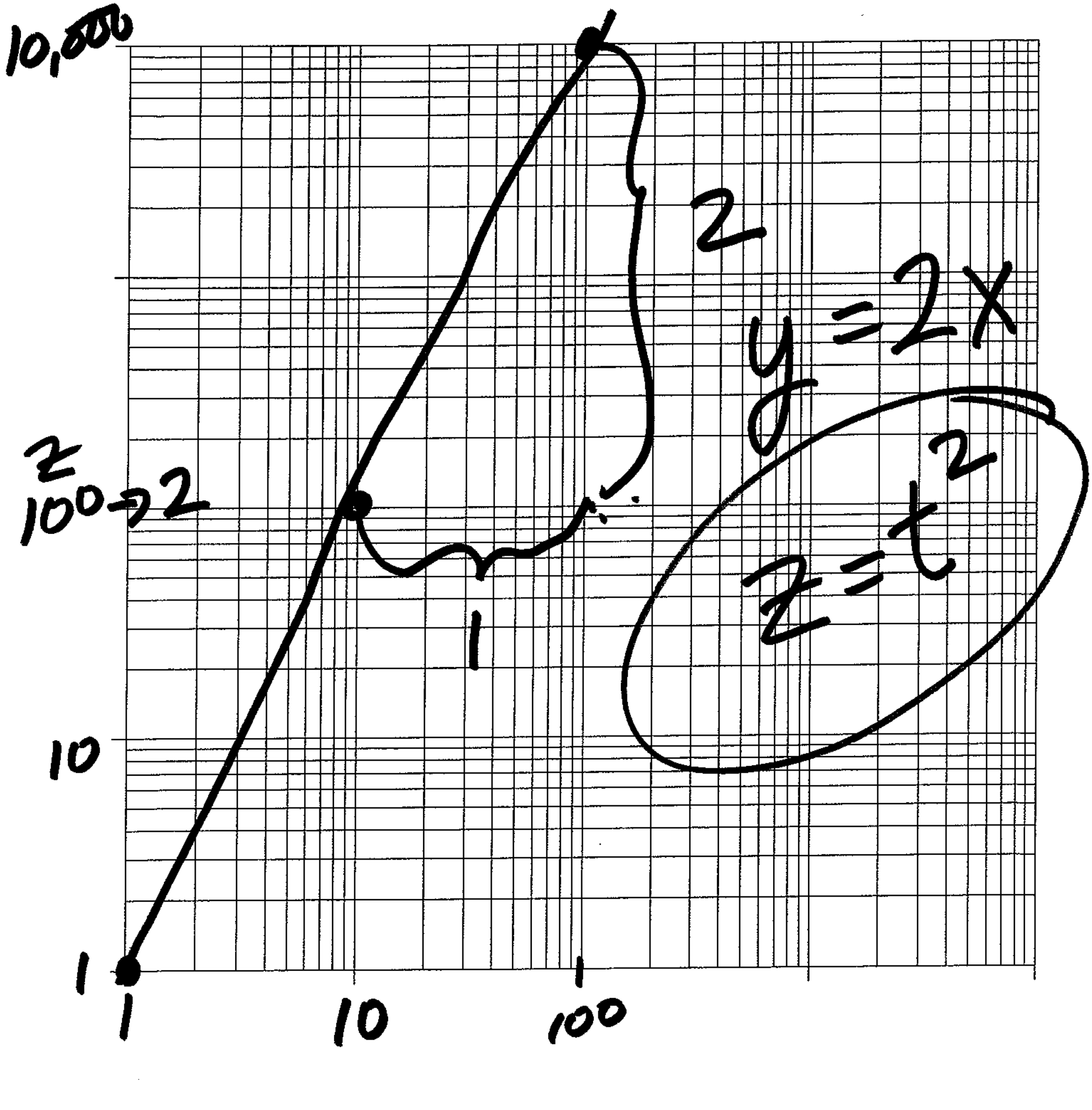
$$z = A t^{m}$$

$$A = 10^{b}$$ ← y-intercept of line

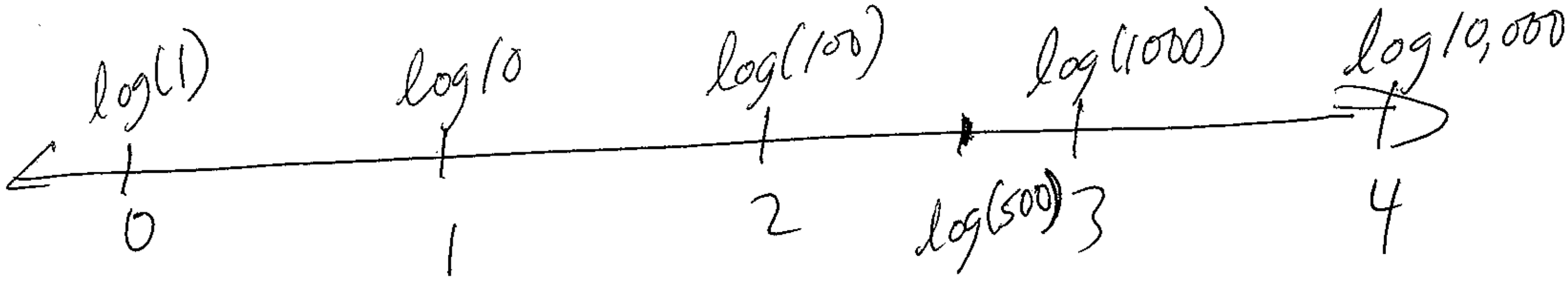
$$m = \text{slope}$$ of line $y = mx + b$

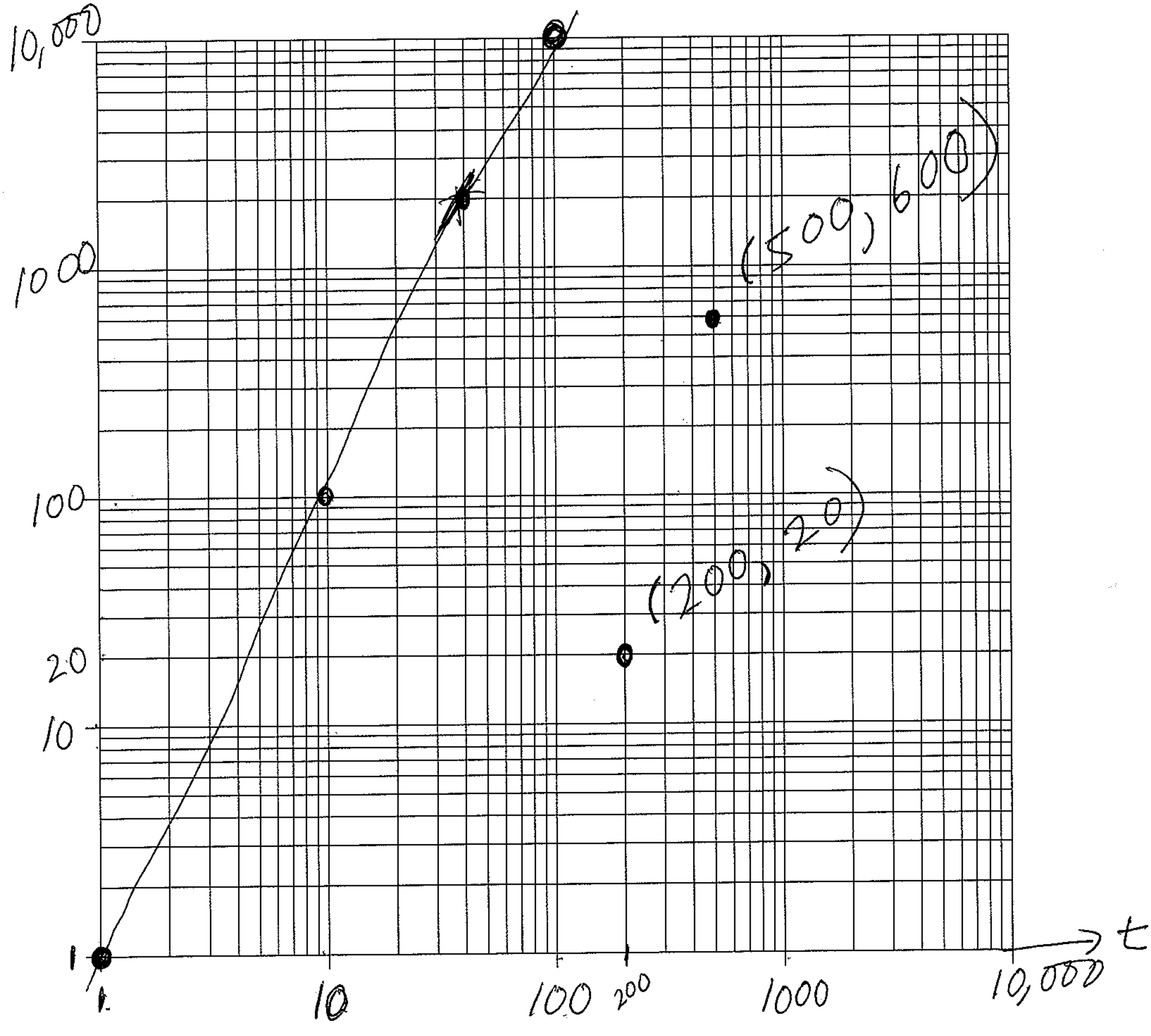


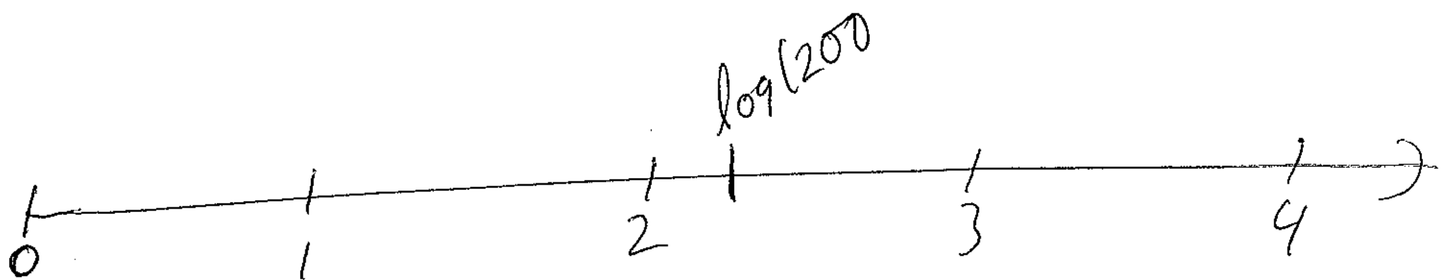
4
3
2
1
0
0
1
2
3
4
y=2x
z=t^2

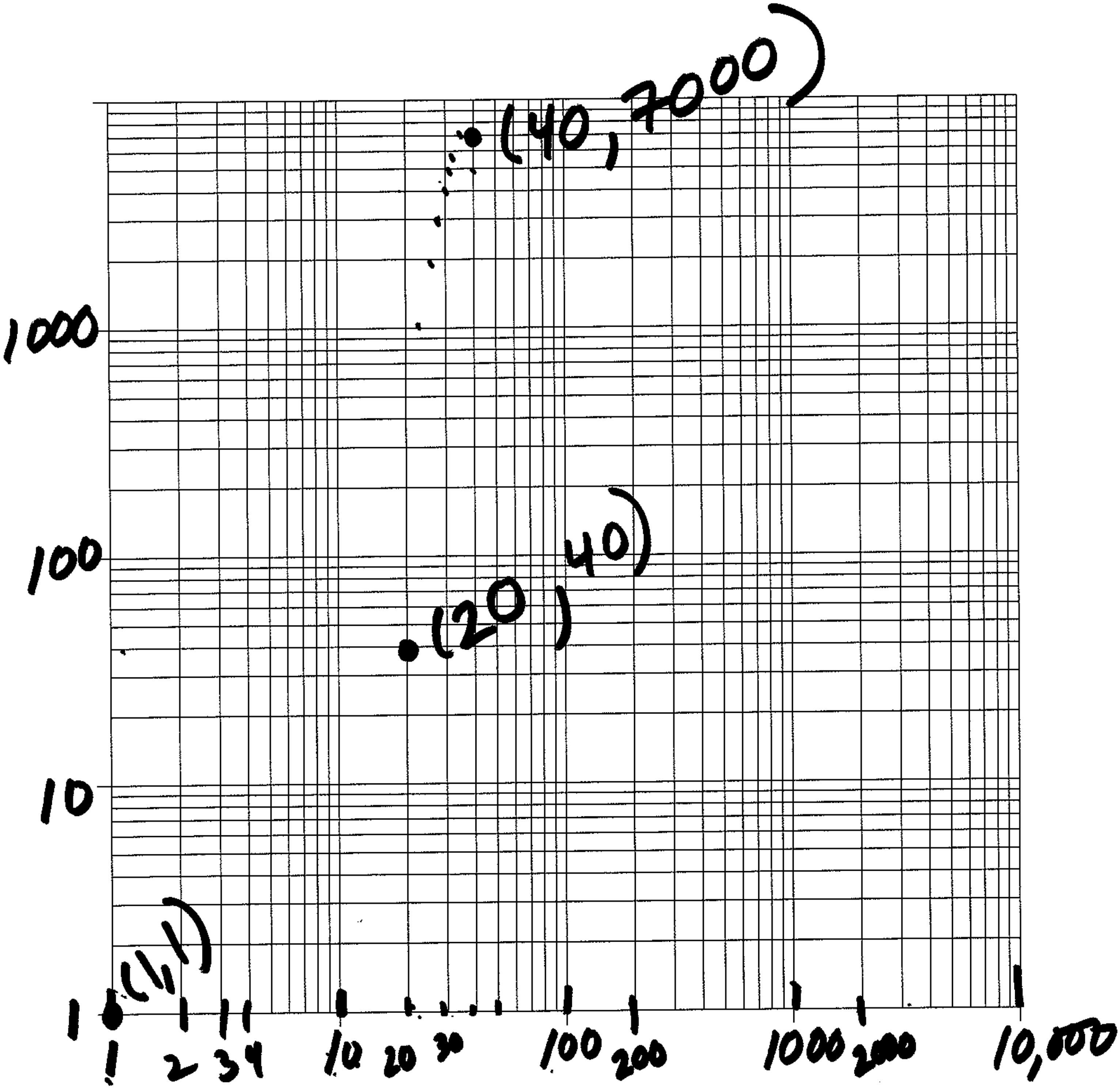


10,000
z
100→2
10
1
1
10
100
2
1
y=2x
z=t^2


log(1)
log10
log(100)
log(1000)
log10,000
0
1
2
log(500)
3
4


10,000
1000
100
20
10
1
(500, 600)
(200, 20)
1
10
100
200
1000
10,000
t


log(200
0
1
2
3
4


(40, 7000)
1000
100
(20, 40)
10
(1,1)
1
1
2
3
4
10
20
30
100
200
1000
2000
10,000