Properties of the definite integral

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$

$\int_a^b (f_1 + f_2)(x)dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$

$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$

If $f_1(x) \leq f_2(x)$, then $\int_a^b f_1(x)dx \leq \int_a^b f_2(x)dx$

If $m \leq f(x) \leq M$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

Find the area between the curve $y^2 = 2x - 2$ and $y = x - 5$.

Use vertical rectangles:

1.) Find points of intersection between the two curves.

$y^2 = 2x - 2$ and $y = x - 5$.

$(x - 5)^2 = 2x - 2$

$x^2 - 10x + 25 = 2x - 2$

$x^2 - 12x + 27 = 0$

$(x - 3)(x - 9) = 0$. Hence $x = 3, 9$.

2.) Determine which is larger.

Between 1 and 3: $\sqrt{2x - 2} > -\sqrt{2x - 2}$

Between 3 and 9: $\sqrt{2x - 2} > x - 5$

3.) Write as integral(s)

Note that between 1 and 3, the height of the rectangles is $\sqrt{2x - 2} - (-\sqrt{2x - 2})$ and the width is dx.

Note that between 3 and 9, the height of the rectangles is $\sqrt{2x - 2} - (x - 5)$ and the width is dx.

$\int_1^3 [\sqrt{2x - 2} - (-\sqrt{2x - 2})]dx + \int_3^9 [\sqrt{2x - 2} - (x - 5)]dx$

4.) Evaluate the integral

$\int_1^3 [2\sqrt{2x-2}]dx + \int_3^9 [\sqrt{2x-2} - (x-5)]dx$

$= \int_1^3 [2\sqrt{2x-2}]dx + \int_3^9 (\sqrt{2x-2})dx - \int_3^9 (x-5)dx$

Let $u = 2x - 2$, $du = 2dx$,
$x = 1 : u = 2(1) - 2 = 0;$
$x = 3 : u = 2(3) - 2 = 4;$
$x = 9 : u = 2(9) - 2 = 16$

$= \int_0^4 u^{\frac{1}{2}} du + \int_4^{16} \frac{1}{2} u^{\frac{1}{2}} du + \int_3^9 (-x+5)dx$

$= \frac{2}{3} u^{\frac{3}{2}} |_0^4 + \frac{1}{3} u^{\frac{3}{2}} |_4^{16} + (-\frac{1}{2}x^2 + 5x)|_3^9$

$$= \frac{2}{3}(4^{\frac{3}{2}} - 0^{\frac{3}{2}}) + \frac{1}{3}(16^{\frac{3}{2}} - 4^{\frac{3}{2}}) + (-\frac{1}{2}(9)^2 + 5(9)) - (-\frac{1}{2}(3)^2 + 5(3))$$

$= \frac{1}{3}[2(8) + 64 - 8] - \frac{81}{2} + 45 + \frac{9}{2} - 15 = 16$

$= \frac{72}{3} - \frac{72}{2} + 30 = 24 - 36 + 30 = 18$

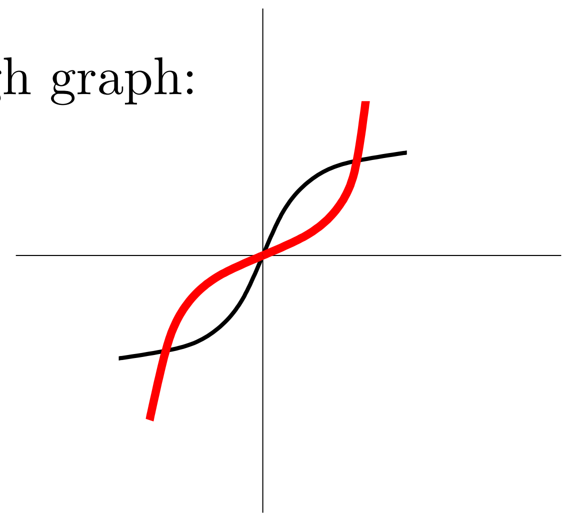
Find the area bounded by the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$.

1.) Find points of intersection:

$2x^3 = 2x^{\frac{1}{3}}$ implies $x^3 = x^{\frac{1}{3}}$ implies $x^9 = x$. Thus $x^9 - x = x(x^8 - 1) = 0$.
Hence $x = 0$ and $x^8 - 1 = 0$. $x^8 = 1$ implies $x = 1, -1$

Hence the functions $y = 2x^3$ and $y = 2x^{\frac{1}{3}}$ intersect when $x = -1, 0, 1$

2.) Draw a rough graph:



3.) Find area:

Use vertical rectangles:

$\int_{-1}^{0}[2x^3 - 2x^{\frac{1}{3}}]dx + \int_0^1 [2x^{\frac{1}{3}} - 2x^3]dx$