Math 150 Exam 1
October 4, 2006

Choose 7 from the following 9 problems. Circle your choices: 1 2 3 4 5 6 7 8 9
You may do more than 7 problems in which case your two unchosen problems can replace your lowest one or two problems at 2/3 the value as discussed in class.

1.) P(10, 7) = $\underline{(10)(9)(8)(7)(6)(5)(4)}$

$C(10,7) = \binom{10}{7} = \underline{\frac{10!}{7!3!} = \frac{(10)(9)(8)}{(3)(2)(1)} = (10)(3)(4) = 120}$

The inversion sequence for the permutation 15243 is $\underline{0, 1, 2, 1, 0}$

The permutation corresponding to the inversion sequence 3, 0, 2, 1, 0 is $\underline{2, 5, 4, 1, 3}$

2.) $r(9,2) = \underline{9}$

$r(3,3) = \underline{6}$

Before $\{x_{13}, x_{12}, x_7, x_1\}$: $\underline{section 4.3}$
After $\{x_{13}, x_{12}, x_7, x_1\}$: $\underline{section 4.3}$
Before $\{2, 8, 13, 14\}$: $\underline{section 4.3}$
After $\{2, 8, 13, 14\}$: $\underline{section 4.3}$

3.) In how many ways can 9 indistinguishable rooks be places on a 20-by-20 chessboard so that no rook can attack another rook?

$\frac{20!}{9!(11)!} \frac{20!}{11!}$

In how many ways can 9 rooks be places on a 20-by-20 chessboard so that no rook can attack another rook if no two rooks have the same color?

$\frac{20!}{9!(11)!} \frac{20!}{11!} 9!$

4.) How many different circular permutations can be made using using 30 beads if you have 20 green beads, 9 blue beads and 1 red beads?

$\frac{29!}{20!\ 9!}$

5.) How many sets of 3 numbers each can be formed from the numbers $\{1, 2, 3, ..., 50\}$ if no two consecutive numbers are to be in a set?

Suppose we think of the 50 numbers as 50 sticks. The number of ways of removing 3 sticks such that no two are consecutive is the same as the number of integral solutions to $x_1 + x_2 + x_3 + x_4 = 47$ where $x_1, x_4 \geq 0$ and $x_2, x_3 \geq 1$. This is the same as the number of solutions to $x_1 + y_2 + 1 + y_3 + 1 + x_4 = 47$ where $x_1, x_4 \geq 0$, $y_2 = x_2 - 1 \geq 1 - 1 = 0$,

$y_3 = x_3 - 1 \geq 1 - 1 = 0$. This is the same as the number of solutions to $x_1 + y_2 + y_3 + x_4 = 45$ where $x_1, x_4, y_2, y_3 \geq 0$.

Hence by thm 3.5.1, the answer is $\binom{45+4-1}{45} = \binom{48}{45} = \frac{48(47)(46)}{6}$

6.) Use the pigeonhole principle to prove that in a group of n people where $n > 1$, there are at least 2 people who have the same number of acquaintances. State where you use the pigeonhole principle.

Number the people 1 through n. We will assume that all acquaintances are mutual. We will also assume that you can't be your own acquaintance. Thus if person i has k_i aquantances among the group of n people, $k_i \in \{0, ..., n-1\}$.

Case 1: There exists someone who knows everyone else. Then $k_i \in \{1, ..., n-1\}$ for $i = 1, ..., n$. Thus by the pigeonhole principle, there exists $i \neq j$ such that $k_i = k_j$.

Case 2: There does not exist someone who knows everyone else. Then $k_i \in \{0, ..., n-2\}$ for $i = 1, ..., n$. Thus by the pigeonhole principle, there exists $i \neq j$ such that $k_i = k_j$.

7.) section 4.5

8.) section 4.5

9.) Use a combinatorial argument to prove $\Sigma_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \binom{2n}{n}$

$\binom{2n}{n}$ = the number of ways to choose n elements from $\{1, ..., 2n\}$.

$\binom{n}{k}$ = the number of ways to choose k elements from $\{1, ..., n\}$.

$\binom{n}{n-k}$ = the number of ways to choose $n-k$ elements from $\{n+1, ..., 2n\}$.

Suppose A is an n-element subset of $\{1, ..., 2n\}$. Let $k = |A \cap \{1, ..., n\}|$.

Thus to choose an n-element subset of $\{1, ..., 2n\}$, we can first fix k and choose k elements from $\{1, ..., n\}$ and $n-k$ elements from $\{n+1, ..., 2n\}$. For a fixed k, the number of ways of choosing k elements from $\{1, ..., n\}$ and $n-k$ elements from $\{n+1, ..., 2n\}$ is $\binom{n}{k}\binom{n}{n-k}$. To get all n element subset of $\{1, ..., 2n\}$, we must do this for $k = 0, ..., n$.

Thus the number of ways to choose n elements from $\{1, ..., 2n\} = \Sigma_{k=0}^{n} \binom{n}{k}\binom{n}{n-k}$.