www.geometrygames.org/TorusGames

Equivalence class $[a] = \{x \mid x \sim a\}$

$\mathcal{P} = \{P_\alpha \mid \alpha \in A\}$ is a partition of X iff
$X = \cup_{P_\alpha \in \mathcal{P}} P_\alpha$, $P_\alpha \neq \emptyset \ \forall \alpha$, and $P_\alpha \cap P_\beta = \emptyset$

Suppose $X = \cup_{\alpha \in B} R_\alpha$ and $R_\alpha \neq \emptyset \ \forall \alpha$, and $R_\alpha \cap R_\beta \neq \emptyset$ implies $R_\alpha = R_\beta$. Then $\mathcal{R} = \{R_\alpha \mid \alpha \in B\}$ is a partition of X

Suppose $a, b \in \mathcal{Z} - \{0\}$. $a \sim b$ if $ab > 0$

$[4] =$

$[-2] =$

Ex: $\mathcal{Z} - \{0\} = \cup_{n \in 2\mathcal{Z} - \{0\}} [n]$

$$= \cup_{n \in \mathcal{Z} - \{0\}} [2n] = (\cup_{n=-1}^{-\infty} [2n]) \cup (\cup_{n=1}^{\infty} [2n])$$

Thm 4.5.3: If $\sim$ is an equivalence relation on X, then
$\{[x_\alpha] \mid x_\alpha \in X\}$ is a partition of X.

If $\mathcal{P} = \{P_\alpha \mid \alpha \in A\}$ is a partition of X, then
$x \sim y$ iff $\exists P_\alpha$ such that $x, y \in P_\alpha$ is an equivalence relation.