Suppose a multiset consisting of integers between 0 and 5 inclusive of size k must contain the following:

even number of 0's: $x^0 + x^2 + x^4 + ... = \frac{1}{1-x^2}$

odd number of 1's: $x^1 + x^3 + x^5 + ... = \frac{x}{1-x^2}$

three or four 2's: $x^3 + x^4 = x^3(1+x)$

the number of 3's is a multiple of five: $x^0 + x^5 + x^{10} + ... = \frac{1}{1-x^5}$

btwn zero to four (inclusive) 4's: $x^0 + x^1 + x^2 + +x^3 + x^4 = \frac{1-x^5}{1-x}$

zero or one 5: $x^0 + x^1 = 1 + x$

$$g(x) = (x^0 + x^2 + x^4 + ...)(x^1 + x^3 + x^5 + ...)(x^3 + x^4)$$
$$(x^0 + x^5 + x^{10} + ...)(x^0 + x^1 + x^2 + +x^3 + x^4)(x^0 + x)$$

$$= \left(\frac{1}{1-x^2}\right)\left(\frac{x}{1-x^2}\right)x^3(1+x)\left(\frac{1}{1-x^5}\right)\left(\frac{1-x^5}{1-x}\right)(1+x)$$

$$= \frac{x^4}{(1-x)^3} = x^4 \Sigma_{k=0}^{\infty} \binom{3+k-1}{k} x^k = \Sigma_{k=0}^{\infty} \frac{(k+2)(k+1)}{2} x^{k+4}$$

Find the number of multisets of size n.

Find the number of multisets of size 100.

Determine the generating function for h_n = the number of ways to make n cents using pennies, nickels, dimes, and quarters.

Note h_n = the number of nonnegative integral solutions to

$$e_1 + 5e_2 + 10e_3 + 25e_4 = n$$

Let $f_1 = e_1$, $f_2 = 5e_2$, $f_3 = 10e_3$, $f_4 = 25e_4$,

Then h_n = the number of nonnegative integral solutions to $f_1 + f_2 + f_3 + f_4 = n$

where f_1 is a nonnegative integer

f_2 is a multiple of 5

f_3 is a multiple of 10

f_4 is a multiple of 25

Hence the generating function for h_n is