

Answers

22M:100 (MATH:3600:0001) Quiz 1
Feb 13, 2013

1.) Match the following differential equations to its direction field:

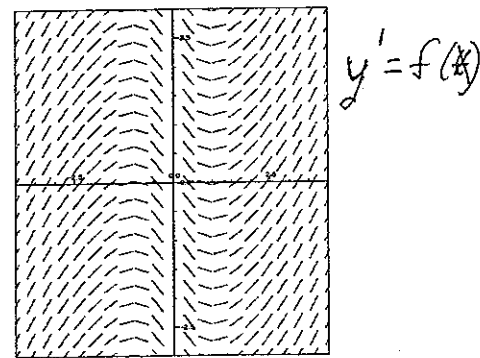
[5] I) $y' = t(t+2)$

[5] II) $y' = y(y+2)$

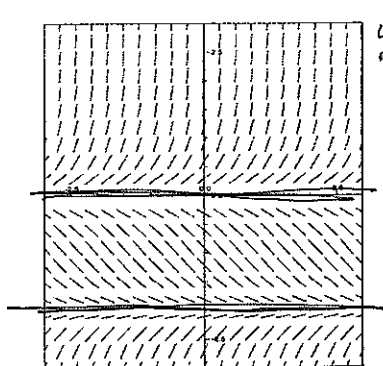
[5] III) $y' = \ln|x|$

[5] IV) $y' = \ln|y|$

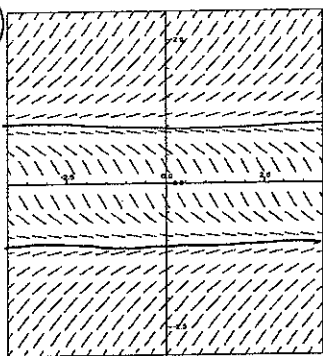
$+ \oplus \ominus \oplus +$



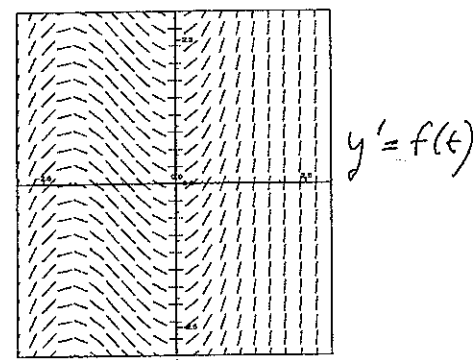
A.) III $y' = \ln|x|$



B.) II $y' = y(y+2)$



C.) IV $y' = \ln|y|$



D.) I $y' = t(t+2)$

[30] 2.) Answer both of the following questions. If your proof to 2A is short, well-written, and correct, you will be given full credit for problem 2. If your answer to 2A is incorrect or if it is not short and well-written (even if correct), your grade for problem 2 will depend solely on 2B.

2A.) Prove that $f : (0, \infty) \rightarrow \mathbb{R}$, $f(x) = \ln(x)$ is 1:1.

Suppose $f(x_1) = f(x_2) \Rightarrow \ln(x_1) = \ln(x_2)$
 $\Rightarrow e^{\ln(x_1)} = e^{\ln(x_2)} \Rightarrow x_1 = x_2 \quad \square$

2B.) Solve $y' = y$.

$\frac{dy}{dt} = y \Rightarrow \int \frac{dy}{y} = \int dt \Rightarrow \ln|y| = t + C$
 $\Rightarrow |y| = e^{t+C} = e^t e^C \Rightarrow y = \pm e^C e^t \Rightarrow y = Ce^t$

Answer 2B: $y = Ce^t$

[50] 3.) Solve the following differential equation (hint: first get it into the appropriate format)

$$y' = \frac{y}{t} + \ln(e)$$

$$\frac{dy}{dt} = \frac{y}{t} + 1 \leftarrow \text{linear, not separable}$$

$$y' - \frac{y}{t} = 1$$

$$-\int \frac{1}{t} dt = -\ln|t| + C; \quad e^{-\ln(t)} = e^{\ln t^{-1}} = t^{-1}$$

$$t^{-1} \left(y' - \frac{y}{t} \right) = 1 \cdot t^{-1}$$

$$t^{-1} y' - t^{-2} y = t^{-1}$$

$$\int (t^{-1} y)' dt = \int t^{-1} dt$$

$$t^{-1} y = \ln|t| + C$$

$$y = t \ln|t| + Ct$$

Answer: $y = t \ln|t| + Ct$